QP CODE: D4BMT2404	(Pages: 3)	Reg. No : ....
		Name : ....

FOURTH SEMESTER FYUGP EXAMINATION, APRIL 2026

Discipline Specific Core (DSC) Courses - Major

MAT4CJ204 : BASIC LINEAR ALGEBRA

(Credits: 4)

Time: 2 Hours | **Maximum Marks: 70**

Section A

Answer the following questions. Each carries 3 marks (Ceiling: 24 marks)

No.	Question	BL	CO
1.	Define Linear Transformation T from a vector space V to a vector space W.Give an Example.	BL1	CO2
2.	Find a basis for $Col\ B$,where $B = \begin{bmatrix} 1 & 4 & 0 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.	BL3	CO2
3.	Identify and describe all possible types of subspaces of $\mathbb{R}^3$.Specify its dimension.	BL1	CO3
4.	Let $A = \begin{bmatrix} 1 & -3 & 4 & -1 & 9 \\ -2 & 6 & -6 & -1 & -10 \\ -3 & 9 & -6 & -6 & -3 \\ 3 & -9 & 4 & 9 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -3 & 0 & 5 & -7 \\ 0 & 0 & 2 & -3 & 8 \\ 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.Given that matrix A is row equivalent to the matrix B. 1. Find $rank\ A$ and $dim\ NulA$. 2. Find basis for $Col\ A,$ and $Row\ A$.	BL3	
5.	If a 3×8 matrix A has rank 3,find $dim\ Nul\ A,\ dim\ Row\ A, and\ rank\ A^T$.	BL2	CO3
6.	Show that if $\mathbf{x}$ is in both W and $W^\perp$, then $\mathbf{x} = \mathbf{0}.$	BL4	CO1
7.	Define an orthogonal set of vectors.Show that $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$ is an orthogonal set, where $\mathbf{u_1} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \mathbf{u_2} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \mathbf{u_3} = \begin{bmatrix} \frac{-1}{2} \\ -2 \\ \frac{7}{2} \end{bmatrix}$.	BL2	CO1

(PTO)

8.	Let $\{\mathbf{v_1}, \mathbf{v_2}\}$ be an orthogonal set of nonzero vectors, and c_1, c_2 be any nonzero scalars. Show that $\{\mathbf{c_1 v_1}, \mathbf{c_2 v_2}\}$ is also an orthogonal set.	BL3	CO1
9.	Let $A = \begin{bmatrix} 4 & 0 \\ 0 & 8 \end{bmatrix}$.Find the singular values of the matrix A.	BL2	CO3
10.	Let $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. Compute $\mathbf{x^T A x}$ for the following matrices: a. $A = \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}$ b. $B = \begin{bmatrix} 3 & -2 \\ -2 & 7 \end{bmatrix}$.	BL3	CO2
	Section B		
	Answer the following questions. Each carries 6 marks (Ceiling: 36 Marks)		
11.	Prove that the Null space of an $m \times n$ matrix A is a subspace of $\mathbb{R}^n$.Equivalently ,the set of all solutions to a system $A\mathbf{x} = \mathbf{0}$ of m homogeneous linear equations in n unknowns is a subspace of $\mathbb{R}^n$.	BL5	CO2
12.	Let H and K be subspaces of a vector space V.The intersection of H and K, written as $H \cap K$, is the set of $\mathbf{v}$ in V that belong to both H and K.Show that $H \cap K$ is a subspace of V.Give an example to show that Union of two subspaces is not,in general ,a subspace.	BL4	CO1
13.	State and prove Unique Representation Theorem.	BL4	CO3
14.	Let H be a nonzero subspace of V ,and suppose T is a one to one linear mapping of V into W. Prove that $dim\ T(H) = dim\ H$.	BL4	CO1, CO3
15.	Find a QR factorization of $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$.	BL3	CO1
16.	Find an orthogonal basis for the column space of the matrix $A = \begin{bmatrix} 3 & -5 & 1 \\ 1 & 1 & 1 \\ -1 & 5 & -2 \\ 3 & -7 & 8 \end{bmatrix}$.	BL3	CO1
17.	Let $\mathbf{u}$ be a unit vector in $\mathbb{R}^n$, and let $\mathbf{B} = \mathbf{uu^T}$. a. Given any $\mathbf{x}$ in $\mathbb{R}^n$, compute $\mathbf{Bx}$ and show that $\mathbf{Bx}$ is the orthogonal projection of $\mathbf{x}$ onto $\mathbf{u}$. b. Show that $\mathbf{B}$ is a symmetric matrix and $\mathbf{B^2} = \mathbf{B}$.	BL4	CO2
18.	Prove that an $n \times n$ matrix A is orthogonally diagonalizable if and only if A is a symmetric matrix.	BL4	CO2

Section C

Answer any one question. Each carries 10 marks (1 x 10 = 10 Marks)

19.	• a) State and prove Spanning Set Theorem. • b) Let $\mathbf{v_1} = \begin{bmatrix} 4 \\ -3 \\ 7 \end{bmatrix}$, $\mathbf{v_2} = \begin{bmatrix} 1 \\ 9 \\ -2 \end{bmatrix}$, $\mathbf{v_3} = \begin{bmatrix} 7 \\ 11 \\ 6 \end{bmatrix}$, and $H = Span\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$. Find a basis for H.	BL5	CO2, CO3
20.	• a) Show that a subset $\{\mathbf{u_1}, \mathbf{u_2}, \ldots, \mathbf{u_p}\}$ in V is linearly independent if and only if the set of coordinate vectors $\{[\mathbf{u_1}]_{\mathcal{B}}, [\mathbf{u_2}]_{\mathcal{B}}, .., [\mathbf{u_P}]_{\mathcal{B}}\}$ is linearly independent in $\mathbb{R}^n$. • b) Given vectors $\mathbf{u_1}, \mathbf{u_2}, .. \mathbf{u_p}$ and $\mathbf{w}$ in V,show that $\mathbf{w}$ is a linear combination of $\mathbf{u_1}, \mathbf{u_2}, .. \mathbf{u_p}$ if and only if $[\mathbf{w}]_{\mathcal{B}}$ is a linear combination of the coordinate vectors $[\mathbf{u_1}]_{\mathcal{B}}, [\mathbf{u_2}]_{\mathcal{B}}, .., [\mathbf{u_p}]_{\mathcal{B}}$.	BL3	CO2

CO : Course Outcome

BL : Bloom's Taxonomy Levels (1 – Remember, 2 – Understand, 3 – Apply, 4 – Analyse, 5 – Evaluate, 6 – Create)