QP CODE: D4BMT2403	(Pages: 2)	Reg. No : ....
		Name : ....

FOURTH SEMESTER FYUGP EXAMINATION, APRIL 2026

Discipline Specific Core (DSC) Courses - Major

MAT4CJ203 : REAL ANALYSIS I

(Credits: 4)

Time: 2 Hours **Maximum Marks: 70**

Section A

Answer the following questions. Each carries 3 marks (Ceiling: 24 marks)

1.	Define (a) an injective (one-one) function. (b) a surjective (onto) function. (c) a bijective function.	BL1	CO1
2.	State Algebraic properties of $\mathbb{R}$.	BL1	CO1
3.	Prove the Uniqueness of Limits of a sequence.	BL1	CO3
4.	Show that the sequence $((-1)^n)$ is divergent.	BL1	CO3
5.	Define a cluster point of a set $A \subset \mathbb{R}$. Interpret the definition of cluster point using neighborhoods.	BL1	CO3
6.	Show that the set $\mathbb{N} \times \mathbb{N}$ is denumerable.	BL1	CO1
7.	Solve the inequality $\lvert 2x - 1 \rvert \leq x + 1$.	BL2	CO1
8.	What is meant by an "ultimately monotone" sequence? Give an example.	BL2	CO3
9.	(a) State the definition of a function f tending to $+\infty$ as $x \to c$. (b) State the definition of a function f tending to $-\infty$ as $x \to c$.	BL1	CO3
10.	(a) State the definition of a function being bounded on a neighborhood of a point. (b) True or False : "If $A \subset \mathbb{R}$ and $f : A \to \mathbb{R}$ has a limit at $c \in \mathbb{R}$, then f is bounded on some neighborhood of c." **(PTO)**	BL1	CO3

Section B

Answer the following questions. Each carries 6 marks (Ceiling: 36 Marks)

11.	(a) State the Principle of Strong Induction. (b) Using Mathematical Induction, prove that $1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$ for all $n \in \mathbb{N}$.	BL1	CO1
12.	(a) State and prove Density Theorem. (b) If x and y are real numbers with $x < y$, then prove that there exists an irrational number z such that $x < z < y$.	BL1	CO2
13.	Let $X = (x_n)$ be a sequence of real numbers that converges to x and suppose that $x_n \geq 0$. Then show that the sequence $(\sqrt{x_n})$ of positive square roots converges and $lim(\sqrt{x_n}) = (\sqrt{x})$.	BL1	CO3
14.	State and prove Monotone Subsequence Theorem.	BL1	CO3
15.	For x_n given by the following formulas, establish either the convergence or the divergence of the sequence $X = (x_n)$. (a) $x_n = \frac{n}{n+1}$. (b) $x_n = \frac{2n^2+3}{n^2+1}$.	BL2	CO3
16.	Suppose that S and T are sets and that $T \subseteq S$. If S is a finite set, then prove that T is a finite set.	BL2	CO1
17.	(a) Define Cauchy sequence. Give an example. (b) If $X = (x_n)$ is a convergent sequence of real numbers, then prove that X is a Cauchy sequence.	BL1	CO3
18.	Let $Y = (y_n)$ be defined inductively by $y_1 = 1; y_{n+1} = \frac{1}{4}(2y_n + 3)$ for $n \geq 1$. Show that $limY = 3/2$.	BL2	CO3

Section C

Answer any one question. Each carries 10 marks (1 x 10 = 10 Marks)

19.	(a) If $I_n = [a_n, b_n], n \in \mathbb{N}$, is a nested sequence of closed, bounded intervals such that the lengths $b_n - a_n$ of I_n satisfy $\inf\{b_n - a_n : n \in \mathbb{N}\} = 0$; then prove that the number ξ contained in I_n for all $n \in \mathbb{N}$ is unique. (b) Prove that the interval $[0, 1]$ is not countable.	BL1	CO2
20.	Show that if $c > 0$, then $\lim(c^{1/n}) = 1$.	BL2	CO3

CO : Course Outcome

BL : Bloom's Taxonomy Levels (1 – Remember, 2 – Understand, 3 – Apply, 4 – Analyse, 5 – Evaluate, 6 – Create)