D4BMT2302 **Reg. No.....**

Name: .....

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2025

(Regular/Improvement/Supplementary)

MATHEMATICS: COMPLEMENTARY COURSE FOR PHYSICS, CHEMISTRY & COMPUTER SCIENCE

GMAT4C04T: MATHEMATICS 4

Time: 2 Hours **Maximum Marks: 60**

SECTION A: Answer the following questions. Each carries *two* marks. (Ceiling 20 marks)

1. State The Continuous Function Theorem for Sequences.
2. Find the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n 5}{4^n}$.
3. Investigate the convergence of the series $\sum_{n=1}^{\infty} \frac{n!}{10^n}$.
4. Find the least upper bound of the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots, \frac{n}{n+1}, \ldots$ if it exists.
5. Find the Maclaurin series for $f(x) = \sin 3x$.
6. Write the formula in the classical Runge-Kutta method of fourth order while solving the initial value problem $y' = f(x, y), y(0) = 1$.
7. Find the Laplace Transform of $\sin 2t cos 3t$.
8. Find the Inverse Laplace Transform of $\frac{5s+1}{s^2-25}$.
9. Verify that $u = x^2 + t^2$ is a solution of the One dimensional wave equation.
10. Find L$(e^{-2t}u(t-3))$.
11. Estimate $\int_1^2 x\, dx$ using Simpson's rule with $n = 4$.
12. Find the radius of convergence and interval of convergence of the series $\sum_{n=1}^{\infty} \frac{3^n x^n}{n!}$.

SECTION B: Answer the following questions. Each carries *five* marks. (Ceiling 30 marks)

13. State the n^{th} Root Test for the convergence of a series of positive terms. Test for convergence of the series $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$.
14. Test for convergence of the sequence, $\{a_n\}$ where $a_n = \frac{n+(-1)^n}{n}$. If convergent, find the limit.
15. Find the Taylor series and Taylor polynomials generated by $f(x) = \cos x$ at $x = 0$.
16. Find the Inverse Laplace Transform, $h(t)$ of $H(s) = \frac{1}{s^2(s^2+a^2)}$ using convolution.
17. Solve the initial value problem $y' = x + y,\ y(0) = 0$ by applying Picard's Iteration.
18. Find the Fourier series expansion of $f(x) = x^2$ in the interval $-\pi < x < \pi$.
19. Find the Laplace Transform of $t^2 sin 3t$.

SECTION C: Answer any *one* question. The question carries *ten* marks.

20. Solve the initial value problem $y'' + 9y = 10e^{-t}, y(0) = 0, y'(0) = 0$.
21. Find the Fourier series expansion of $f(x) = \begin{cases} x & if\ -\pi < x < 0. \\ \pi - x & if\ \ 0 < x < \pi \end{cases}$

    in the interval $-\pi < x < \pi$.

(1 × 10 = 10 Marks)