D4BMT2301 **Reg. No.............................**

Name: ....

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2025

(Regular/Improvement/Supplementary)

MATHEMATICS

GMAT4B04T: MULTIVARIABLE AND VECTOR CALCULUS II

Time: 2 ½ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries *two* marks.
(Ceiling 25 marks)

1. If z = f (x,y), where x=u-v and y=v-u, show that $\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v} = 0$.
2. Find gradient of $f = xyz$.
3. Find the directional derivative of $f(x, y) = e^x \cos 2y$ at $(0, \pi/4)$ in the direction of v =2 i + 3 j.
4. Find the equation of the normal line to the curve $x^2 - y^2 = 16$ at (5,3).
5. Find the critical points of $f(x, y) = x^2 + 3y^2 - 6xy - 2x + 4y$.
6. Evaluate $\int_0^2 \int_{-1}^2 (2x + 3xy^2)\ dx\ dy$.
7. How will you find the volume of a solid region using triple integrals?
8. Evaluate $\int_0^1 \int_0^2 \int_1^2 x^2yz\ dz\ dy\ dx$.
9. Find the divergence of $\vec{F} = xy^2\hat{i} + yz^2\hat{j} + zx^2\hat{k}$.
10. State Green's theorem for plane regions.
11. Find the curl of $e^{-x}\cos y\ \hat{\imath} + e^{-x}\sin y\ \hat{\jmath} + In(z)\ \hat{k}$.
12. State fundamental Theorem for line integrals.
13. Find the Jacobian J(u,v) of the transformation *u*=*x*+*2y* and *v*=*x*-*y*.
14. Find the equation of the tangent plane to the curve $z = 9x^2 + 4y^2$ at (-1,2,25).
15. Write the parametric equations of a sphere.

SECTION B: Answer the following questions. Each carries *five* marks
(Ceiling 35 marks)

16. Find a vector giving the direction in which the function f decreases most rapidly. What is the maximum rate of decrease ,where $f(x, y) = \tan^{-1}(2x + y),\ P(0, 0)$.

(PTO)

17. Find and classify the relative extrema and saddle points of $f(x,y) = xy(3-x-y)$.

18. Find the maximum and minimum distance from the origin to the curve:

$5x^2 + 6xy + 5y^2 - 10 = 0$

19. Reverse the order and evaluate $\int_0^1 \int_y^1 sin(x^2)\ dx\ dy$.

20. Evaluate $\iint_R (2x+3y)\ dA$ where R is the region in the first quadrant bounded by the circles $x^2+y^2=1$ and $x^2+y^2=4$.

21. Find the volume of the tetrahedron with vertices (0,0,0),(1,0,0),(0,3,0) and (0,0,2).

22. Find an equation of the tangent plane to the surface **r**(u,v) = (u+v) **i** + (u-v) **j** + v^2 **k,** at (2,0,1).

23. Evaluate $\int_C xy\ dx - xy^2\ dy$, where C is the quarter circle from (0,-1) to (1,0) centered at the origin.

SECTION C: Answer any *two* questions. Each carries *ten* marks.

24. Find the surface area of the part of the sphere $x^2+y^2+z^2=8$ that lies inside the cone $z^2=x^2+y^2$.

25. Find the centroid of a homogeneous solid hemisphere of radius a.

26. Let $\vec{F} = 2xy^2z^3\ \hat{\imath} + 2x^2yz^3\hat{\jmath} + 3x^2y^2z^2\ \hat{k}$. Show that $\vec{F}$ is conservative, and find the potential function f such that $\vec{F} = \nabla f$. Also find the Work done by $\vec{F}$ in moving a particle along any path from (0,0,0) to (1,1,1).

27. Use Stoke's theorem to evaluate $\iint_S Curl\ \vec{F}.$ dS given $\vec{F} = 2y\hat{\imath} + xz^2\hat{\jmath} + x^2ye^z\hat{k}$ and S is the hemisphere $z = \sqrt{4 - x^2 - y^2}$ oriented with normal pointing outward.

(2 × 10 =20 Marks)