D4BHM2302 **PAGES: 3** **Reg.No..............................**

Name: ....

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2025

Regular/Supplementary/Improvement

HONOURS IN MATHEMATICS

GMAH4B16T - CALCULUS IV

Time: 3 Hours **Maximum Marks: 80**

SECTION A: Answer all the questions. Each carries 1 mark

1. $\nabla f(P)$ is ...... to the level curve $f(x, y) = c$ at a point P on it.
   (A) parallel (B) orthogonal
   (C) tangential (D) asymptotic.

2. The equation of the normal line at the point $(2, 1, -3))$ to the surface $2x^2 + y^2 + 2z = 3$ is ....
   (A) $8(x - 2) + 2(y - 1) + 2(z + 3) = 0$ (B) $\frac{x-2}{4} = y - 1 = z + 3$
   (C) $4(x - 2) + (y - 1) + (z + 3) = 0$ (D) none of these.

3. At the point $(0, 0, 0)$ the function $f(x, y) = y^2 - x^2$ has a
   (A) relative maximum (B) relative minimum
   (C) absolute extremum (D) saddle point.

4. At the point $(0, 0)$ the function $f(x, y) = x^2 + 2y^2 + x^2y + 3$ has a
   (A) relative maximum (B) relative minimum
   (C) saddle point (D) none of these.

5. The value of $\int_0^2 \int_1^4 y\sqrt{x}dydx$ is
   (A) $2\sqrt{2}$ (B) $\sqrt{2}$ (C) $2\sqrt{2} - 1$ (D) none of these.

6. If $R = \{(x, y)|\, 0 \le y \le 1, y \le x \le 2y\}$, then $\iint\limits_R (1 + 2x + 2y)dA = \ldots\ldots$

7. The equivalent form of $\iiint\limits_T f(x, y, z)dV$ in spherical coordinates is ......

8. The gradient vector field of the scalar function $f(x, y, z) = x^2y - y^3$ is ......

9. The divergence of $\mathbf{F}(x, y) = \frac{1}{x+1}\mathbf{i}$ is ......

(PTO)

10. If C is the curve $r(t) = t\ \mathbf{i} + 4t\ \mathbf{j}, 0 \leq t \leq 1$, then $\int_C (x+y)ds = \ldots\ldots$

(10 x 1 = 10 marks)

SECTION B: Answer any 8 questions. Each carries 2 marks

11. Define normal line

12. State Lagrange's theorem on multipliers.

13. What is meant by an x-simple region

14. Evaluate $\int_0^{\frac{\pi}{2}} \int_0^1 r^5 dr d\theta$

15. What is meant by an r-simple region.

16. Evaluate the integral $\int_0^{2\pi} \int_1^2 \int_0^{2-r} r\ dz\ dr\ d\theta$

17. Find the Jacobian of the transformation T defined by the equations

$$x = e^u \cos 2v \quad , \quad y = e^u\ \sin 2v$$

18. A vector field $\mathbf{F}$ in $R^2$is defined by $\mathbf{F}(x, y) = x\mathbf{i} +\ y\mathbf{j}$. Describe $\mathbf{F}$ and sketch a few vectors representing the vector field.

19. Find the curl of $\mathbf{F}(x, y, z) = x\mathbf{i}+y\mathbf{j}$

20. Find a parametric representation for the cone $x^2 + y^2 = z^2$

(8 x 2 = 16 marks)

SECTION C: Answer any 6 questions. Each carries 4 marks

21. Find equations of the tangent plane and normal line to the surface $x^2 + 4y^2 + 9z^2 = 17$ at the point $(2, 1, 1)$.

22. Find the volume of the solid lying under the elliptic paraboloid $z = 8 - 2x^2 - y^2$ and above the rectangular region $R = \{(x, y)|0 \leq x \leq 1,\ 0 \leq y \leq 2\}$.

23. Evaluate the integral by changing to polar coordinates $\iint_R xydA$ where R is the region in the first quadrant bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = y$

24. Find the centroid of a homogeneous solid hemisphere of radius a.

25. Find the divergence and curl of the vector field $\mathbf{F} = yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}$

26. Evaluate $\int_C y\ dx + x^2 dy$, *where*

    a) C is the line segment C_1 from $(1, -1)$ *to* $(4, 2)$

    b) C is the arc C_2 of the parabola $x = y^2$ from $(1, -1)$ to $(4, 2)$

    c) C is the arc C_3 of the parabola $x = y^2$ from $(4, 2)$ to $(1, -1)$

27. Determine whether the vector field $\mathbf{F}(x, y, z) = yz\ \mathbf{i} + xz\ \mathbf{j} + xy\ \mathbf{k}$ is conservative.

    If so, find a function f such that $\mathbf{F} = \nabla f$.

28. Let T be the solid bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $z = 3$, and let S be the surface of T. calculate the outward flux of the vector field $\mathbf{F}(x, y, z) = xy^2\ \mathbf{i} + yz^2\ \mathbf{j} + zx^2\ \mathbf{k}$ over S

(6 x 4 = 24 marks)

SECTION D: Answer any 2 questions. Each carries 15 marks

29. Find the absolute maximum and the absolute minimum values of the function $f(x, y) = 2x^2 + y^2 - 4x - 2y + 3$ on the rectangle $D = \{(x, y) : 0 \leq x \leq 3, 0 \leq y \leq 2\}$.

30. a). Find the Jacobian of the transformation T defined by the equations $x = u + v + w,\ y = u - v + w\ , z = u - 2v + 3w$.

    b). Evaluate $\iint_R (x + y)dA$, where R is the parallelogram bounded by the lines with equations $y = -2x, y = \frac{1}{2}x - \frac{15}{2}, y = -2x + 10$ and $y = \frac{1}{2}\ x$;

    using the transformation T defined by $x = u + 2v$ and $y = v - 2u$

31. Let $\mathbf{F}$ be a continuous vector field in an open, connected region R. Prove that the line integral $\int_C \mathbf{F}.d\mathbf{r}$ is independent of path if and only if $\mathbf{F}$ is conservative

(2 x 15 = 30 marks)