D4BHM2304 (PAGES 4) **Reg. No.............................**

Name: ....

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2025

(Regular/Improvement/Supplementary)

HONOURS IN MATHEMATICS

GMAH4B18T: NUMERICAL COMPUTING

Time: 3 Hours **Maximum Marks: 80**

Part A: Answer all the questions. Each carries *one* mark.

Choose the correct answer.

1. The iterative formula in Newton Raphson method is __________.

   a) $x - \frac{f(x)}{f'(x)}$ c) $x - \frac{f'(x)}{f(x)}$

   b) $x + \frac{f(x)}{f'(x)}$ d) None of the above

2. Newton- Gregory Forward interpolation formula can be used ______________.

   a) only for equally spaced intervals.

   b) only for unequally spaced intervals.

   c) for both equally and unequally spaced intervals.

   d) for unequally intervals.

3. If $f(t, y)$ satisfies the Lipschitz conditions in y then the Lipschitz constant for $f(t, y) = ysint,\ 0 \leq t \leq 1$ is:

   (a) $L = 0$ (b) $L = 1$ (c) $L = 2$ (d) $L = 3$

4. The approximate value of $y(0.1)$ from $\frac{dy}{dx} = x^2y - 1,\ 0 \leq x \leq 1,\ y(0) = 1$ is:

   (a) 0.900 (b) 1.001 (c) 0.802 (d) 0.994

5. Euler's method is Taylor's method of order ________.

   (a) zero (b) one (c) two (d) three

Fill in the blanks.

6. In bisection method, for $f(x) = x^3 - 3x + 1$ on [1,2] , $P_2 =$ ________
7. Using Newton Raphson formula for $f(x) = 2x^3 + 2x - 6 = 0$, $P_0 = 1, P_2$ _______.
8. If $f(3) = 5,\ f(5) = 3,\ f(7) = 9,$ then $\Delta(f(3))$ is ________.
9. The Newton's divided difference formula is ________.
10. The maximum error occurred when using the backward-difference formula in numerical differentiation is _______.

(10 x 1 = 10 Marks)

(PTO)

Part B: Answer any *eight* questions. Each carries *two* marks.

11. Use the Bisection method to find P_3 for $f(x) = \sqrt{x} - \cos x$ on $[0, 1]$
12. Describe the trapezoidal method.
13. Determine any fixed points of the function $g(x) = x^2 - 2$.
14. Let $f(x) = x^2 - 3x$ and $P_0 = 1$. Use Newton's method to find P_2.
15. Determine the linear Lagrange interpolating polynomial that passes through the points $(2, 4)$ and $(5, 1)$.
16. Use the forward-difference formula to approximate the derivative of $f(x) = ln\, x$ at $x_0 = 1.8$ using $h = 0.01$
17. For the following data, find $f'(1.0)$.

x	$f(x)$	$f'(x)$
1.0	1.0000	
1.2	1.2625	
1.4	1.6595	

18. Calculate $\int_0^2 (x+1)^{-1}\, dx$ using Simpsos's rule.
19. Given $3\frac{dy}{dx} + 5y^2 = sinx$, $y(0.3) = 5$ and using a step size of $h = 0.3$, find the value of $y(0.9)$ using Euler's method.
20. Write down the formula for the Modified Euler method.

(8 x 2 = 16 Marks)

Part C: Answer any *six* questions. Each carries *four* marks.

21. Use the Bisection method to find P_5 for $f(x) = x^3 - 7x^2 + 14x - 6 = 0$ on $[1, 3.2]$
22. Find a root of $x^3 + 4x^2 - 10 = 0$, accurate to within 10^{-2} using fixed point method. Use $P_0 = 1$.
23. Let $f(x) = x^2 - 6$, with $P_0 = 3$ and $P_1 = 2$,find P_2 using
    a) Secant method
    b) Method of false position
    c) Which of **a)** or **b)** is closer to $\sqrt{6}$.
24. Approximate f (0.65) using the following data and the Newton backward-difference formula:

x	0.0	0.2	0.4	0.6	0.8
$f(x)$	1.00000	1.22140	1.49182	1.82212	2.22554

25. Find a bound for the error in calculating $\int_0^{0.5} \frac{2}{x-4}\ dx$ using Simpson's rule and compare this to the actual error.
26. Use composite trapezoidal rule to calculate $\int_0^2 \frac{2}{x^2+4}\ dx$, $n = 6$
27. Show that there is a unique solution to the initial value problem $y' = cost,\ 0 \le t \le 1,\ y(0) = 1$ and hence find the solution.
28. Use Runge Kutta method of order four to approximate the solution for the initial-value problem, $\mathrm{y}' = \cos 2\mathrm{t} + \sin 3\mathrm{t},\ 0 \le \mathrm{t} \le 1, \mathrm{y}(0) = 1,$ with $h = 0.5$

(6 x 4 = 24 Marks)

Part D: Answer any *two* questions. Each carries *fifteen* marks.

29. a) For $f(x) = \cos x$, let $x_0 = 0, x_1 = 0.6\ and\ x_2 = 0.9$, construct interpolating polynomial of degree two to approximate $f(0.45)$ and find the absolute error.

b) Construct divided difference table for the following data

x	$f(x)$
1.0	0.7651977
1.3	0.6200860
1.6	0.4554022
1.9	0.2818186
2.2	0.1103623

30. Given the initial-value problem $y' = \frac{2}{t}y + t^2e^t, 1 \le t \le 2, y(1) = 0$, with exact solution $y(t) = t^2(e^t - e)$. Use Euler's method with $h = 0.1$ to approximate the solution and compare it with the actual values of y.

(PTO)

31. a) Use the most accurate three-point formula to determine each missing entry in the following table.

x	$f\ (x)$	$f'(x)$
1.1	9.025013	
1.2	11.02318	
1.3	13.46374	
1.4	16.44465	

b) Determine the values of n and h required to approximate $\int_0^2 x^2 \cos x \; dx$ to within 10−4. Use:

i) Composite Trapezoidal rule.

ii) Composite Simpson's rule.

iii) Composite Midpoint rule.

(2 x 15 = 30 Marks)