D4BEM2302 (PAGES 2) **Reg. No.............................**

Name: ....

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2025

(Regular/Improvement/Supplementary)

ECONOMICS & MATHEMATICS (DOUBLE MAIN)

GDMT4A02T: THEORY OF EQUATIONS AND COMPLEX NUMBERS

Time: 2 ½ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries *two* marks.

(Ceiling 25 marks)

1. If a and b are different and $f(x)$ is separately divisible by $x-a$ and $x-b$ show that $f(x)$ is divisible by $(x-a)(x-b)$.
2. Expand x^3-1 in powers of $x-1$.
3. State the fundamental theorem of algebra.
4. What is the relation between p and q if the equation $x^3+px+q=0$ has a multiple root.
5. Factorize x^4+4 into linear factors.
6. Solve the cubic equation $2x^3-x^2-18x+9=0$ whose roots are $a, b,$ and c and $a+b=0$.
7. Find the upper limit of the positive roots of the equation $x^4-7x^3+10x^2-30=0$.
8. Show that the equation $x^3-7x+7=0$ has a real roots within the interval [-4, -3].
9. Separate the roots of the equation $3x^4-74-6x^2+12x-1=0$.
10. State Descarte's rule of signs.
11. If α, β and γ are the roots of the equation $x^3-7x+7=0$, find the equation whose roots are $\alpha^2+\beta^2;\ \beta^2+\gamma^2;\alpha^2+\gamma^2$.
12. Describe the set of points z in the complex plane that satisfy $|z| = |z-i|$.
13. Use De Moivre's formula find trigonometric identities for $\cos 2\theta$ and $\sin 2\theta$.
14. Find all solutions of the equation $z^4+1=0$.
15. Find the image of the square S with vertices at $1+i, 2+i, 2+2i, and\ 1+2i$ under the linear mapping $T(z) = z+2-i$.

(PTO)

SECTION B: Answer the following questions. Each carries *five* marks.

(Ceiling 35 marks)

16. Without actual division show that $2x^4 - 7x^3 - 2x^2 + 13x + 16$ is divisible by $x^2 - 5x + 6$.
17. Using Horner's method expand $x^4 - 6x^3 + x^2 - 1$ in powers of $x + 1$.
18. Find the rational roots of the polynomial equation $3x^3 - 7x^2 - 5x + 2 = 0$.
19. Prove that the rational roots of the equation $x^n + a_1x^{n-1} + \cdots . a_n = 0$; with integer coefficients can be only integers.
20. Verify that the function $(x_1 + x_2)(x_1 + x_3)\ (x_2 + x_3)$ is symmetric and break them up into sigma functions.
21. Prove that between two consecutive roots c and d d of $f'(x)$ lies at most one root of f(x).
22. Find the natural domain and the range of the complex functions $f(z) = \frac{z+\bar{z}}{z-\bar{z}}$.
23. Find the image of the circle $|z - z_0| = R$ under the mapping $f(z) = iz - 2$ using parametrizations.

SECTION C: Answer any *two* questions. Each carries *ten* marks.

24. Explain the synthetic division procedure to find the quotient and remainder when dividing $f(x)$ by $(x - c)$.
25. a) Find the highest common divisor of $x^4 - 6x^2 - 8x - 3$ and $x^3 - 3x - 2$.

    b) Find an upper limit of the moduli of roots for the equation $x^3 - 6x^2 + 11x - 6 = 0$.
26. For what values of A has the equation $(x + 3)^3 - A(x - 1) = 0$ three real roots.
27. a) Find the image of the set S defined by $|z| \leq 2,\ 0 \leq arg(z) \leq \pi/2$, under the mapping $w = z^3$.

    b) Prove that for any two complex numbers z_1 and z_2 ,

    $arg(z_1z_2) = arg(z_1) + arg(z_2)$ and $arg\ (\frac{z_1}{z_2}) = arg(z_1) - arg(z_2)$.

(2 × 10 = 20 Marks)