89

D4BMT1802 (S4) (PAGES 2) Reg. No..............................

Name: ....

FOURTH SEMESTER B. Sc. DEGREE EXAMINATION, APRIL 2024

(Supplementary - 2018 Admission)

MATHEMATICS: Complementary Course for Chemistry, Physics & C.S

AMAT4C04T: MATHEMATICS - 4

Time: 3 Hours **Maximum Marks: 80**

PART A: Answer *all* the questions. Each carries 1 mark.

1. Find $L^{-1}(te^{t})$.
2. Show that $y = e^{x}$ is a solution of the differential equation $y'' - y = 0$.
3. Find $L(2t + 6)$.
4. Define a trigonometric series.
5. Find $L^{-1}(\frac{1}{(s+1)^2})$.
6. Write the error estimate for Simpson's Rule.
7. Find $L^{-1}(\frac{a}{s^2-a^2})$.
8. Write the general form of one dimensional wave equation.
9. Draw the graph of $f(x) = x, -\pi < x < \pi$.
10. Check whether the function $f(x) = x\ sinx$ is odd or even.
11. Give an example of a partial differential equation.
12. Find the Wronskian of e^{x} and xe^{x}.

(12 × 1 = 12 Marks)

PART B: Answer any *nine* questions. Each carries 2 marks.

13. Solve $y'' + y = 0$.
14. Write the Eulers formula for finding Fourier coefficients.
15. Find $L(\cosh at)$.
16. If $f(t) = sin^2 t$, find $L(f(t))$.
17. Check whether $u = x^2 + t^2$ is a solution of the wave equation.
18. Find $L^{-1}(\frac{5s}{s^2-25})$.
19. Solve $y'' + y' - 2y = 0$.
20. Define a Fourier Cosine Series.
21. Solve $u_{xy} = -u_x$.
22. Find $L\left(\frac{\sin at}{t}\right)$.

(PTO)

23. Find the fundamental period of $\cos 2x$

24. Apply Picards Iteration to $y' = y$, y(0)=1, for computing numerical values of solutions.

(9 × 2 = 18 Marks)

PART C: Answer any *six* questions. Each carries 5 marks.

25. Solve $y'' + y = \sec x$

26. Find $L^{-1}(\frac{6}{(s+2)(s-4)})$.

27. Solve $y'' - y = t, y(0) = 1\ ;\ y'(0) = 1$

28. Find the Inverse Laplace transform of $\ln(1 + \frac{w^2}{s^2})$.

29. Solve $x^2y'' - 4xy' + 6y = 0$

30. Find the two half range expansions of the function $f(x) = 1, 0 < x < L$.

31. Solve $u_x + u_y = 0$.

32. Find the solution of the wave equation corresponding to the triangular initial deflection

$$f(x) = \begin{cases} \frac{2k}{L}x, & 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L - x), & \frac{L}{2} < x < L \end{cases}$$, and initial velocity zero.

33. Solve $y' = y, y(0) = 1$ *and* $h = 0.1$ using Improved Euler Method.

(6 × 5 = 30 Marks)

PART D: Answer any *two* questions. Each carries 10 marks.

34. Determine the response of the damped mass spring system governed by

    $y'' + 3y' + 2y = r(t), y(0) = 0;\ y'(0) = 0$ where r(t) is

    a) the square wave $r(t) = u(t-1) - u(t-2)$.

    b) the unit impulse at time t=1, $r(t) = \delta(t-1)$.

35. Find the Fourier Series of the function $f(x) = 3x(\pi^2 - x^2);\ -\pi < x < \pi$.

36. a) Use Trapezoidal rule with n=4 to estimate $\int_1^2 x^2 dx$. Find an upper bound for error in the approximation for the value of $\int_1^2 x^2 dx$.

    b) Using Simpsons Rule with n=4,estimate the integral $\int_1^3 (2x-1)dx$.

(2 × 10 = 20 Marks)