D4BMT2202 **Reg.No.............................**

Name: ....

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2024

(Regular/Improvement/Supplementary)

MATHEMATICS: COMPLEMENTARY COURSE FOR PHYSICS, CHEMISTRY & COMPUTER SCIENCE

GMAT4C04T: MATHEMATICS 4

Time:2Hours **MaximumMarks: 60**

SECTION A: Answer the following questions. Each carries *two* marks.

(Ceiling 20 Marks)

1. Find $\lim_{n\to\infty} \frac{2^n}{5n}$
2. Find the radius of convergence and interval of convergence of the series $\sum_{n=1}^{\infty} \frac{3^n x^n}{n!}$.
3. Find the Maclaurin series for $f(x) = \frac{1}{1+x}$.
4. Write the first shifting property of Laplace transform.
5. Find the Laplace Transform of $cos^2 at$.
6. Find the Inverse Laplace Transform of $\frac{12}{(s-3)^4}$.
7. Investigate the convergence of the series $\sum_{n=1}^{\infty} n!\, e^{-n}$.
8. Verify that $u = e^{-t} sinx$ is a solution of the One dimensional heat equation.
9. Estimate $\int_{-1}^{1} (x^2 + 1\,)dx$ using trapezoidal rule with $n = 4$.
10. Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.
11. Write the formula in the classical Runge-Kutta method of fourth order while solvingthe initial value problem $y' = f(x, y), y(0) = 1$.
12. Find L($e^{-2t} u(t-3)$).

SECTION B: Answer the following questions. Each carries *five* marks.

(Ceiling 30 Marks)

13. Test for convergence of the sequence, $\{a_n\}$ where $a_n = \frac{1-2n}{1+2n}$. If convergent, find the limit.
14. State the Ratio Test for the convergence of a series of positive terms. Test for convergence of the series $\sum_{n=0}^{\infty} \frac{2^n+5}{3^n}$.
15. Find the Taylor series and Taylor polynomials generated by e^x at $x = 0$.
16. Find the Inverse Laplace Transform,$h(t)$ of $H(s) = \frac{1}{s(s^2+4)}$ using convolution.
17. Find the Laplace Transform of $t^2 cos\omega t$.
18. Find the half range cosine series of $f(x) = x^2$ in the interval $0 < x < \pi$.
19. Solve the initial value problem $y' = y, y(0) = 1$ by applying Picard's Iteration.

SECTION C: Answer any *one* question.Each carries *ten* marks.

20. Solve the initial value problem $y'' - y = t, y(0) = 1, y'(0) = 1$.
21. Find the Fourier series expansion of $f(x) = x + x^2$ in the interval $0 < x < 2\pi$.

(1 x 10 = 10 Marks)