D4BMT2201 (PAGES:2) **Reg.No.............................**

Name: ....

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2024

(Regular/Improvement/Supplementary)

MATHEMATICS

GMAT4B04T: MULTIVARIABLE AND VECTOR CALCULUS II

Time: 2 ½ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries *two* marks.
(Ceiling 25 marks)

1. $w = x^2 + xy + y^2 + z^3, x = 2t, y = e^t, z = cos2t$. Find $\frac{dw}{dt}$.
2. Find gradient of $f = x^2yz$.
3. Find the directional derivative of $f(x,y) = \frac{x+y}{x-y}$ at (2,1) in the direction of $v = i + 3j$.
4. Find the equation of the normal line to the curve $2x + y - e^{x-y} = 2$ at$(1,1)$.
5. Find the critical points of $f(x,y) = x^2 + y^2 - 4x - 6y + 17$.
6. Evaluate $\int_0^1 \int_1^2 (3x^2y)\, dx\, dy$.
7. State Fubini's theorem for rectangular regions.
8. Evaluate $\int_1^e \int_1^e \int_1^e \frac{1}{xyz} dz\, dy\, dx$.
9. Find the divergence of $\vec{F} = xycosy\hat{\imath} + ysinx\hat{\jmath} + zx\hat{k}$.
10. State vector form of Green's theorem.
11. Find the curl of $x^2y^3\hat{\imath}$+$xz^2\hat{k}$.
12. What is an oriented surface? Give an example of a non-oriented surface.
13. Find the Jacobian *J(u,v)* of the transformation *u=x-y* and *v=2x+y.*
14. Find the equation of the tangent plane to the curve $z = 9x^2 + 4y^2\ at\ (-1,2,2,5)$.
15. Write the parametric equations of a cone.

SECTION B: Answer the following questions. Each carries *five* marks.
(Ceiling 35 marks)

16. Find a vector giving the direction in which the function f increases most rapidly. What is the maximum rate of increase of $f(x,y) = \sqrt{xy}\cos z, P(4,1,\pi/4)$.
17. Find and classify the relative extrema and saddle points of $f(x,y) = x^2 - 6x - x\sqrt{y} + y$.
18. Find 3 positive real numbers whose sum is 300 and product is as large as possible.
19. Use double integrals to find the area enclosed by one loop of the 3 petalled rose, $r = sin3\theta$.

(P.T.O.)

20. Evaluate $\iint_R \sqrt{x^2+y^2}\,dA$, where R is the region in the first quadrant bounded by the circles $x^2+y^2=4$ and the lines$y=0$ and$y$=√3 x.

21. Find the volume of the solid S that lies below the hemisphere $z=\sqrt{9-x^2-y^2}$above the x-y plane and the cylinder $x^2+y^2=1$.

22. State and prove Green's theorem for simple regions.

23. Evaluate$\int_C yz\,dx - y\cos x\,dy + y\,dz$,

where C is the curve $x=t$, $y=\cos t$, $z=\sin t$, $0\le t\le \frac{\pi}{2}$.

SECTION C: Answer any *two* questions. Each carries *ten* marks.

24. Find the surface area of the part of the plane $2x+3y+z=12$ that lies above the rectangular region R={(x,y)| 0 ≤ x ≤ 2, 0 ≤ y ≤1}.

25. Find the mass and centre of mass of the lamina, occupying the region D in the first quadrant bounded by the circle$x^2+y^2=1, p(x,y)\sqrt{x^2+y^2}$.

26. Let $\overrightarrow{F}$=$yz^2\hat{\imath}$+ $xz^2\hat{\jmath}$+ 2xyz $\hat{k}$. Show that $\vec{F}$ is conservative and find the potential function f such that $\overrightarrow{F}$=∇f. Also find the work done by $\vec{F}$ in moving a particle along any path from (0,0,1) to (1,3,2).

27. Compute $\iint_S \vec{F}\cdot\vec{n}\,dS$ given that $\overrightarrow{F}$(x,y,z)=(x+sinz) $\hat{\imath}$ +(2y+cosx) $\hat{\jmath}$ +(3z+tany) $\hat{k}$

and S is the unit sphere x^2+y^2+z^2=4.

(2 × 10 =20 Marks)