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D4BMC2202 (PAGES 2) Reg.No............................

Name: ....

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2024

COMPUTER SCIENCE & MATHEMATICS (DOUBLE MAIN)

GDMA4A02T: THEORY OF EQUATIONS AND COMPLEX NUMBERS

Time: 2 ½ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carry *two* marks.

(Ceiling 25 Marks)

1. By the method of detached coefficients multiply $x^5 - 3x^4 + x^3 - x + 1$ by $3x^3 - 7x^2 - x + 1$.
2. Divide $3x^5 + 2x^4 - 5x^3 + x^2 - 4x + 6$ by $x^2 - 2x + 3$.
3. Show that the square of an integer divided by 3 results in either 0 or 1 as the remainder.
4. Show that $x = 1$ is the only multiple roots of the equation $x^n - nx + n - 1 = 0$.
5. Using Taylor formula expand $\frac{1}{x}$ in powers of $(x - 1)$.
6. How many real roots has the equation $x^4 - 4ax + b = 0$.
7. Find the sum of squares of the roots of the equation $2x^4 - 6x^3 + 5x^2 - 7x + 1 = 0$.
8. Find the upper limit of the positive roots of the equation $x^4 - 7x^3 + 10x^2 - 30 = 0$.
9. Show that the equation $x^3 - 2x - 5 = 0$ has a real root m within the interval [1,3].
10. State Rolles's theorem in Calculus.
11. Define symmetric function and give an example.
12. If α, β and γ are the roots of the equation $x^3 - 3x + 1 = 0$, find the equation whose roots are $\alpha + \alpha^{-1}$; $\beta + \beta^{-1}$; $\gamma + \gamma^{-1}$.
13. Show that the complex function $f(z) = z + 3i$ is one-to-one on the entire complex plane and find a formula for its inverse function.
14. For the complex numbers $z_1 = -1$ and $z_2 = 5i$, show that $$Arg(z_1 . z_2) \neq Arg(z_1) + Arg(z_2).$$
15. Express $-\sqrt{3} - i$ in polar form.

SECTION B: Answer the following questions. Each carry *five* marks.

(Ceiling 35 Marks)

16. State and prove the remainder theorem.
17. Determine k and solve the equation $2x^4 - 15x^3 + kx^2 - 30x + 8 = 0$, if the roots are in geometric progression.
18. Find the rational roots of the polynomial equation $3x^3 - 7x^2 - 5x + 2 = 0$.
19. Solve the biquadratic equation $x^4 + \quad 4x - 1 = 0$ using Ferrari's method.
20. Prove that the equation $x^3 - 6x^2 + 9x - 4 = 0$ has at least two real roots.
21. Prove that complex roots of polynomial with real coefficients occur in conjugate pairs with same multiplicity.
22. Find the four fourth roots of $z = 1 + i$.
23. Suppose the product $z_1 z_2$ of two complex numbers is a nonzero real constant. Show that $z_2 = k\bar{z_1}$, where k is a real number.

(PTO)

SECTION C: Answer any *two* questions. Each carry *ten* marks.

24. Solve the cubic equation $x^3 - 3x^2 + 1 = 0$ trigonometrically.

25. a) Find the highest common divisor of $2x^4 + 2x^3 - 3x^2 - 2x + 1$ and $x^3 + 2x^2 + 2x + 1$.

    b) State and prove the identity theorem.

26. a) Find the image of the set S defined by $|z| \leq 3, \pi/2 \leq arg(z) \leq 3\pi/4$, under the principal square root function.

    b) Find an upper bound for $\frac{-1}{z^4 - 5z + 1,}$ if |z|=2.

27. For what values of A will the equation $(x + 3)^3 - A(x - 1) = 0$ possess three real roots?

(2 x 10 = 20 Marks)