D4BHM2204 (PAGES 3) Reg.No.....

Name: .....

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2024

HONOURS IN MATHEMATICS

GMAH4B18T: NUMERICAL COMPUTING

Time: 3 Hours **Maximum Marks: 80**

PART A: Answer all the questions. Each carries *one* mark.

Choose the correct answer.

1. What is the bisection method used for in numerical analysis?
   a) Matrix inversion. b) Finding roots of equations.
   c) Integration. d) Interpolation.
2. Lagrange interpolation formula can be used _____
   a) only for equally spaced intervals. b) only for unequally spaced intervals.
   c) for both equally and unequally spaced intervals. d) for unequally intervals.
3. Let $f(x) = x^2 - 3x + 1 = 0$ *and* $P_0 = 0$. Then value of P_2 using Newton Raphson method is.....
   a) 0.290164 b) -0.290164 c) 0.213703 d) None of the above.
4. The error bound when approximating $\int_a^b f(x)dx$ using Trapezoidal rule is
   a) $\frac{h}{12} f''(\xi)$ b) $\frac{h^2}{12} f''(\xi)$ c) $\frac{h^3}{12} f''(\xi)$ d) None of the above.
5. Which of the following is a solution to y ' = 6x, where y = 2 for x = 0?
   a) $y = 2x^3 + 2$ b) $y = 2x^3$ c) $y = 3x^2 + 2$ d) $y = 3x^2$

Fill in the Blanks.

6. Using Newton Raphson formula for $f(x) = x^3 + 4x - 6 = 0$, $P_0 = 1, P_2$ _____
7. The backward-difference formula in numerical differentiation is _____
8. The Trapezoidal rule for approximating $\int_a^b f(x)dx$ is _____
9. The Newton's forward interpolation formula is _____
10. The error bound for the approximation of $f(x)$ with a polynomial of degree using Lagrange interpolation formula is _____

(10 x 1 = 10 Marks)

(PTO)

PART B: Answer any *eight* questions. Each carries *two* marks.

11. Define Lipschitz condition for a function in $\mathcal{R}^2$and give an example.
12. Use the forward-difference formula to approximate the derivative of $f(x) = \ln x$ at $x_0 = 1.8$ using $h = 0.05$
13. Determine the linear Lagrange interpolating polynomial that passes through the points $(4, 2)$ *and* $(3, 1)$.
14. Construct the forward difference table for the following data:

x	$f(x)$
0	0
2	2
4	15
6	20
8	25
10	33

15. If $(0) = 0$, $f(1) = 3$ *and* $f(2) = 8$, find $f(1.5)$.
16. Calculate $\int_0^2 e^x \, dx$ using Trapezoidal rule.
17. Use the Bisection method to find P_3 for $f(x) = 2x^3 - 3x - 5$ *on* $[0, 2]$
18. Use Euler's method to approximate the solution to $y' = y - t^2 + 1, 0 \leq t \leq 2, y(0) = 0.5$, at $t = 2$ with $h = 0.5$.
19. Define polynomials. Give an example.
20. Write down the formula for Runge Kutta Fourth order method.

(8 x 2 = 16 Marks)

PART C: Answer any *six* questions. Each carries *four* marks.

21. Find an approximation to $\sqrt{3}$ using Bisection algorithm.
22. Find the second Lagrange interpolating polynomial for $f(x) = \sin x$ with nodes $x_0 = 0, \quad x_1 = 1, \quad x_2 = 2$
23. Find a bound for the error in calculating $\int_0^1 x^2 \, e^{-x} \, dx$ using Trapezoidal rule and compare this with the actual error.
24. A fourth-degree polynomial P(x) satisfies $\Delta^4 P(0) = 24$, $\Delta^3 P(0) = 6$ *and* $\Delta^2 P(0) = 0$ where $\Delta P(x) = P(x + 1) - P(x)$. Compute $\Delta^2 P(4)$
25. Use the forward-difference formulas and backward-difference formulas to determine each missing entry in the following table.

x	$f(x)$	$f'(x)$
0.5	0.4794	
0.6	0.5646	
0.7	0.6442	

26. Find the error bound when approximating $f(x) = \ln(x+1)$ by a polynomial of degree two with $x_0 = 0, x_1 = 0.6$ *and* $x_2 = 0.9$ at $x = 0.45$

27. Given $3\frac{dy}{dx} + \sqrt{y} = e^{0.1x}, y(0.3) = 5$ and using a step size of h = 0.3, find the best estimate of $\frac{dy}{dx}(0.9)$ using Euler's method.

28. Use the Midpoint method with $N = 4, h = 0.5, t_i = 0.5i$, and $w_0 = 0.5$ to approximate the solution of, $y' = y - t^2 + 1, 0 \leq t \leq 2, y(0) = 0.5$.

(6 x 4 = 24 Marks)

PART D: Answer any *two* questions. Each carries *fifteen* marks.

29. Find a root of $f(x) = 3x\,e^x = 0$ in [1 , 2] using:
    a) Newton Raphson method.
    b) Secant method.
    c) Method of false position.

30. a) Use the most accurate three-point formula to determine each missing entry in the following table.

x	$f(x)$	$f'(x)$
2.0	3.6887983	
2.1	3.6905701	
2.2	3.6688192	
2.3	3.6245909	

b) Find $\int_0^{\frac{\pi}{4}} \sin x \; dx$ using closed and open Newton-Cotes formulas.

31. Use Modified Euler's method to approximate the solutions for the initial-value problem, $y' = \frac{2-2ty}{t^2-1}, 0 \leq t \leq 1, y(0) = 1,$ with $h = 0.1$.

(2 x 15 = 30 Marks)