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D4BHM2203 (PAGES 4) **Reg. No..............................**

Name: ....

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2024

HONOURS IN MATHEMATICS

GMAH4B17T: LINEAR PROGRAMMING AND APPLICATIONS

Time: 3 Hours **Maximum Marks: 80**

PART A: Answer *all* the questions. Each carries *one* mark.

Choose the correct answer.

1. The closed ball of radius $r > 0$ centered at the origin in $\mathbb{R}^1$ is................

   A) Circle. B) Sphere. C) Disc. D) line segment.

2. An unconstrained linear programming problem.

   A) Has no nonnegativity constraint. B) Has nonnegativity constraint.

   C) has many constraints. D) has only one constraint.

3. For any pair of feasible solutions of dual canonical linear programming problem.

   A) $g \geq f$ B) $g \leq f$ C) $g = f$ D) $g \neq f$

4. In a matrix game any mixed strategy containing an entry of 1 is called...............

   A) probabilistic strategy. B) pure strategy.

   C) column strategy. D) row strategy.

5. Solution of the given transportation problem is.................

12	10	8	28
8	9	11	62
20	38	22	

   A)786 B) 678 C) 876 D) 687

Fill in the Blanks.

6. Canonical slack maximization linear programming problem is...............
7. Basic solution is obtained by ....
8. "Any unbounded linear programming problem has an unbounded constraint set".The Statement is..............(True/False)
9. The duality equation is..........
10. "If a canonical minimization linear programming problem is infeasible, then the dual canonical maximization linear programming problem is unbounded." The Statement is...........(True/False)

(10 x 1 = 10 Marks)

(PTO)

PART B: Answer any *eight* questions. Each carries *two* marks.

11. Draw and shade a bounded polyhedral convex subset in $\mathbb{R}^2$.
12. Define hyperplane. Give an example.
13. Pivot on 5 in the canonical maximum tableau given below.

x_1	x_2	-1	
1	2	3	$=-t_1$
4	5	6	$=-t_2$
7	8	9	$=f$

14. If a canonical maximization linear programming problem is unbounded, prove that the dual canonical minimization linear programming problem is infeasible.
15. State duality theorem.
16. Consider the canonical maximization linear programming problem given below :

    Maximize $f(x_1, x_2) = x_1$

    $x_1 + x_2 \leq 1$

    $x_1 - x_2 \geq 1$

    $x_2 - 2x_1 \geq 1 \qquad x_1, x_2 \geq 0$

    State the dual canonical minimization of the linear programming problem.
17. State von Neumann mini-max theorem.
18. Let $x, y \in \mathbb{R}$ and consider the matrix game given below:

$$\begin{array}{cc} & \text{II} \\ \text{I} & \begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix} \end{array}$$

    Determine a necessary and sufficient condition for the matrix game above to reduce by Domination to a single entry.
19. Explain briefly the North West corner method to obtain the initial basic feasible solution in transportation problem.
20. What is an assignment problem.

(8 x 2 = 16 Marks)

PART C: Answer any *six* questions. Each carries *four* marks.

21. Apply the simplex algorithm to solve the maximum tableau

x_1	x_2	-1	
1	2	20	$=-t_A$
2	2	30	$=-t_B$
2	1	25	$=-t_C$
200	150	0	$=P$

22. Show that the given LPP is infeasible

$$\text{Maximize } f(x,y) = 3x + 2y$$

$$\text{Subject to } 2x - y \leq -1$$

$$x - 2y \geq 0$$

$$x, y \geq 0$$

23. Solve the noncanonical linear programming problem given below:

Maximize $f(x,y,z) = x - y + z$

subject to $x + y \geq 2$

$z - y \geq 3$

$2x + z \leq 8$

24. Write the dual simplex algorithm for minimum tableaus.

25. Solve the following dual canonical linear programming problem:

	x_1	x_2	-1	
y_1	1	-1	-1	$= -t_1$
y_2	-1	-1	-1	$= -t_2$
-1	1	-2	0	$= f$
	$= s_1$	$= s_2$	$= g$	

26. Prove that a pair of feasible solutions of dual canonical linear programming problem exhibit complementary slackness if and only if they are optimal solutions.

27. Solve the transportation problem given below:

	M_1	M_2	M_3	
W_1	2	1	2	40
W_2	9	4	7	60
W_3	1	2	9	10
	40	50	20	110

28. Solve the assignment problem given below:

38	21	34
41	14	36
28	20	25

(6 x 4 = 24 Marks)

(PTO)

PART D: Answer any *two* questions. Each carries *fifteen* marks.

29. Solve the canonical linear programming problem given below using the simplex algorithm.

x	y	z	-1	
1	2	1	4	$= t_1$
2	1	5	5	$= t_2$
3	2	0	6	$= t_3$
1	2	3	0	$= f$

30. Find the von Neumann value and optimal strategy for each player in the matrix game given below.

$$\begin{array}{c} \\ \text{I} \\ \\ \end{array} \overset{\text{II}}{\begin{bmatrix} -1 & 0 & 2 & -2 & 0 \\ 1 & -2 & -4 & 2 & 2 \\ 0 & -1 & 1 & 1 & -1 \\ 0 & 5 & 4 & 2 & 0 \end{bmatrix}}$$

31. Using Hungarian algorithm solve the following assignment problem:

4	6	5	10
10	9	7	13
7	11	8	13
12	13	12	17

(2 x 15 = 30 Marks)