67

D4BHM2202 (PAGES 2) Reg. No..............................

Name: ....

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2024

HONOURS IN MATHEMATICS

GMAH4B16T: CALCULUS IV

Time: 3 Hours **Maximum Marks: 80**

PART A: Answer all the questions. Each carries *one* mark.

Choose the correct answer.

1. The function $f(x, y) = x^3 - 3xy^2$ has critical points at..............
   A) (0,0) only B) (1,-1) and (-1,1) only
   C) (1,1) and (-1,-1) only D) (0,0), (1, -1) and (-1,1)
2. What is the primary purpose of triple integrals?
   A) Calculating the volume under a surface in three dimensions.
   B) Finding the area under a curve in two dimensions.
   C) Measuring the length of a curve.
   D) Evaluating a line integral.
3. Which of the following statements about the Jacobian determinant is true?
   A) Always negative. B) A scalar value.
   C) Vector quantity. D) Independent of choice of co-ordinates.
4. The curl of the vector field $\mathcal{F}(x,y,z) = y\mathbf{i}$ for $x \geq 0$ is ....
   A) k B) -k C) 2k D) 0
5. A surface S which has a unit normal vector that varies continuously over S is called ....
   A) Parametric surface B) Orientable surface C) Smooth surfacee D) Rough Surface

Fill in the Blanks.

6. $\nabla f(a,b)$ is ....to the level curve $f(x, y) = c$ at (a, b).
7. If $f(x, y) \geq f(a,b)$ for all points in the domain of f, then f has a ....at (a,b).
8. If g is defined in a region R in the xy-plane, then the area of the surface $y = g(x,z)$ is A = ----
9. The divergence of $\mathcal{F} = P\mathbf{i}+Q\mathbf{j}+R\mathbf{k}$ in three dimensional space is defined by div $\mathcal{F}$= ....
10. The flux of a vector field $\mathcal{F}$ across an oriented surface in the direction of the unit normal **n** is..............

(10 x 1 =10 Marks)

PART B: Answer any *eight* questions. Each carries *two* marks.

11. Find the level curve of $f(x, y) = x^2 - y^2$ passing through the point (5,3).
12. Explain the second derivative test for relative extrema.
13. State Lagrange's theorem.
14. Evaluate $\iint_R 2\, dA$, where R = [-1,3] x [2,5].
15. State Fubini's theorem for rectangular regions.

(PTO)

16. Find the area of the surface S which is the part of the plane $2x + 3y + z = 12$ that lies above the rectangular region $R = \{(x,y) / 0 \leq x \leq 2, 0 \leq y \leq 1\}$.
17. A vector field $\mathcal{F}$ in R^2 is defined by $\mathcal{F}(x,y) = -y\mathbf{i} + x\mathbf{j}$. Describe $\mathcal{F}$ and sketch few vectors representing the vector field.
18. What is a conservative vector field? Give an example.
19. Find the work done on a particle that moves along the quarter circle of radius 1 centered at the origin in a counter clockwise direction from (1,0) to (0,1) by the force field $\mathcal{F}(x, y, z) = -y\mathbf{i} + x\mathbf{j} + z\mathbf{k}$.
20. Find a parametric representation for the cone $x^2 + y^2 = z^2$.

(8 x 2 = 16 Marks)

PART C: Answer any *six* questions. Each carries *four* marks.

21. Find the equations of the tangent plane and normal line to the ellipsoid with equation $4x^2 + y^2 + z^2 = 16$ at the point $(1,2,\sqrt{2})$.
22. Evaluate $\int_0^1 \int_0^x \int_0^{x+y} x \, dz \, dy \, dx$.
23. Evaluate $\iint_R (1 - 2xy^2) \, dA$, where $R = \{(x, y) / 0 \leq x \leq 2, -1 \leq y \leq 1\}$.
24. Find the volume of the solid that lies below the hemisphere $z = \sqrt{9 - x^2 - y^2}$, above the xy-plane, and inside the cylinder $x^2 + y^2 = 1$.
25. Evaluate $\int_C xydx - yzdy + x^2dz$, where C consists of the line segment from (0,0,0) to (1,1,0) and the line segment (1,1,0) to (2,3,5).
26. Find the area of that part of the plane $y + z = 2$ inside the cylinder $x^2 + z^2 = 2$.
27. Show that $\mathcal{F}(x,y,z) = 2xyz^2\mathbf{i} + x^2z^2\mathbf{j} + 2x^2yz\mathbf{k}$ is conservative and find a function f such that $\mathcal{F} = \nabla f$.
28. Find the flux of the vector field $\mathcal{F}(x, y, z) = y\,\mathbf{i} + x\,\mathbf{j} + 2z\,\mathbf{k}$ across the unit sphere $x^2 + y^2 + z^2 = 1$.

(6 x 4 = 24 Marks)

PART D: Answer any *two* questions. Each carries *fifteen* marks.

29. Find the maximum and minimum values of the function $f(x, y) = x^2 - 2y$ subject to $x^2 + y^2 = 9$.
30. Evaluate $\iint_R \cos\left(\frac{x-y}{x+y}\right) dA$, where R is the trapezoidal region with vertices (1,0),(2,0),(0,2) and (0,1).
31. a) Find the gradient vector field of the function $f(x, y, z) = \frac{-k}{\sqrt{x^2+y^2+z^2}}$ and hence deduce that the inverse square field $\mathcal{F}$ is conservative.

    b) Evaluate $\iint_S \frac{x-y}{\sqrt{2z+1}} dS$, where S is the surface represented by $\mathbf{r}(u, v) = (u + v)\mathbf{i} + (u - v)\mathbf{j} + (u^2 + v^2)\mathbf{k}$ where $0 \leq u \leq 1$ and $0 \leq v \leq 2$.

(2 x 15 = 30 Marks)