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D4BHM2201 (PAGES 2) **Reg. No..............................**

Name: ....

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2024

HONOURS IN MATHEMATICS

GMAH4B15T: REAL ANALYSIS II

Time: 3 Hours **Maximum Marks: 80**

PART A: Answer all the questions. Each carries *one* mark.

Choose the correct answer.

1. Which of the following functions are continuous on [0,1]

   A) $\frac{1}{x}$ B) $\frac{1}{e^x}$ C) $\frac{1}{\sqrt{x}}$ D) cot x

2. Every convergent sequence is....................

   A) absolutely convergent. B) monotone. C) oscillating. D) bounded.

3. The value of $\int_0^1 x\, dx$

   A) 0 B) 1 C) $\frac{1}{2}$ D) $\frac{1}{\sqrt{2}}$

4. Value of $\lim_{n\to\infty} \frac{nx}{1+(nx)^2}$.

   A) 0 B) 1 C) $\frac{1}{2}$ D) -1

5. The sum of the infinite series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ is ....

   A) 0 B) 1 C) $\frac{1}{2}$ D) -1

Fill in the Blanks.

6. The maximum value of $x^2 + 10$ in [-1,1] is ....
7. The norm of partition P = (0,1.5,2,3.4,4) of [0,4] is...........
8. Value of $\lim(x^n)$ for $x \in (-1,1)$ is ....
9. The formula for the series $\sum_{n=1}^{\infty} r^{2n}$ when |r| < 1 is..............
10. $\int_{-5}^{5} sgn(x)dx$ = ..., where $sgn(x)$ denotes the Signum Function.

(10 x 1 = 10 Marks)

PART B: Answer any *eight* questions. Each carries *two* marks.

11. Define Lipchitz function. Give an example.
12. Show that that the function $f(x) = \sin(\frac{1}{x}), x \neq 0$ is not continuous at 0.
13. Show that $g(x) = \frac{1}{x}$ is not uniformly continuous on A =[0,1].
14. Show that if $\sum a_n$ is absolutely convergent, then it is convergent.

(PTO)

15. Evaluate $\int_1^9 \frac{\sin\sqrt{t}}{\sqrt{t}}\, dt$.

16. Show that $f(x) = k, \forall x \epsilon [a, b]$ is Riemann integrable.

17. State true or false: Every bounded function is Riemann integrable. Justify your answer.

18. Let $f_n : [0,1] \rightarrow \mathbb{R}$ be a sequence of bounded functions. i.e, $\forall n, \exists M_n$ such that $|f_n(t)| < M_n, \forall t \epsilon [0,1]$. If (f_n) uniformly converges to f, Show that f is bounded on [0, 1].

19. Check whether (f_n) where $f_n(x) = \frac{x}{n}\ \forall x \epsilon \mathbb{R}$ is uniformly convergent.

20. Find the first and second partial sum of the series $\sum \frac{1}{2^{n-1}}$.

(8 x 2 = 16 Marks)

PART C: Answer any *six* questions. Each carries *four* marks.

21. State and prove Bolzano's intermediate value theorem.

22. If $f: I \rightarrow \mathbb{R}$ is continuous on I = [a.b],a closed and bounded interval, show that f is bounded on I.

23. Show that the equation $x = \cos x$ has a solution in the interval [0,2].

24. Let g: $[0,3] \rightarrow \mathbb{R}$ be defined by $g(x) = 1$ for $0 \leq x \leq 1$ and $g(x) = 2$ for $1 < x \leq 3$. Find $\int_0^3 g$.

25. State and prove Fundamental theorem of Calculus first form.

26. Show that $\sum \frac{\cos\theta}{n^p}$ is uniformly convergent for all real values of $\theta, p > 1$.

27. Prove that sequence (f_n) of bounded functions on $A \subseteq \mathbb{R}$ converges uniformly on A to f if and only if $\|f_n - f\|_A \rightarrow 0$.

28. Show that the sequence $\left(\frac{x^n}{(1+x^n)}\right)$ does not converge uniformly on [0, 2] by showing that the limit function is not continuous on [0, 2].

(6 × 4 = 24 Marks)

PART D: Answer any *two* questions. Each carries *fifteen* marks.

29. State and prove Maximum Minimum theorem.

30. State and prove additivity theorem of Riemann integral.

31. Let (f_n) be a sequence of functions in R[a, b] and suppose that (f_n) converges uniformly on [a,b] to f. Show that $f \epsilon R[a, b]$ and $\int_a^b f = \lim_{n\rightarrow\infty} \int_a^b f_n$.

(2 x 15 = 30 Marks)