D4BEM2204

Reg. No............................

Name: ..............................

**FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2024**
**ECONOMICS & MATHEMATICS**
GDMT4B04T-ABSTRACT ALGEBRA

Time: 2.5 Hours | Total : 80 Marks

**SECTION A**: Answer the following questions. Each carries **two** marks
(Ceiling 25)

1. Define Equivalence relation on a set $S$.
2. Consider the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 2 & 7 & 6 & 3 & 8 & 1 & 4 \end{pmatrix}$. Represent $\sigma$ as the product of disjoint cycles.
3. Find the inverse of the cycle $\sigma = (2, 5, 8, 6)$ in $S_8$.
4. Define even permutation. Give example.
5. Give example of a non-abelian group.
6. Construct addition table for $Z_4$.
7. Show that if every element of a group $G$ is its own inverse, then $G$ is abelian.
8. Find $\phi(24)$.
9. Find all generators of $Z_{15}$.
10. State the necessary and sufficient condition for a homomorphism of a group $G_1$ to $G_2$ to be an isomorphism.
11. Find all cosets of $4Z$ in $2Z$
12. Define simple group Give example.
13. Compute the factor group $\frac{(Z_6 \times Z_4)}{< [(2,3)] >}$.
14. State True or false : Every Ring is an integral domain. Justify.
15. State Fundamental homomorphism theorem

**SECTION B**: Answer the following questions. Each carries **five** marks
(Ceiling 35)

16. Show that $S_n$ the set of all permutations of $S = \{1, 2, , \cdots, n\}$, has $n!$ elements.
17. Show that the set $G = \{a + b\sqrt{2} : a, b \in Q\}$, where $Q$ is the set of all rational numbers, is an abelian group with respect to addition.
18. Show that if $H$ and $K$ are two subgroups of group $G$, $H \cap K$ is also a subgroup of $G$. Whether $H \cup K$ a subgroup of $G$? Justify your answer.

19. Find order of $(1, 2, 5)(2, 3, 4)(5, 6)$.
20. Show that every subgroups of a cyclic group is cyclic.
21. Define a homomorphism from $Z \to Z_n$. Verify
22. If $G$ is finite group and $H$ a subgroup of $G$, then $[G : H] = \frac{o(G)}{o(H)}$.
23. Show that the cancellation laws for multiplication hold in a commutative ring $R$ if and only if $R$ has no zero divisors.

**SECTION C**: Answer any **two** question.( 2 x 10=20 Marks)

24. Show that every permutation in $S_n$ can be written as a product of cycles. The cycles, of length greater than or equal to two, that appear in the product are unique.
25. State and Prove Lagranges theorem. Also prove that for $a \in G$, a finite group of order $n$, $o(a)|n$ and $a^n = e$.
26. Find all subgroups of $Z_{18}$ and draw the subgroup diagram.
27. State and prove Cayley's Theorem.