D4BMT2101 (PAGES 2) Reg.No............................

14

Name: ....

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2023

(Regular/Improvement/Supplementary)

MATHEMATICS

GMAT4B04T: MULTIVARIABLE AND VECTOR CALCULUS II

Time: 2 ½ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries *two* marks.
(Ceiling 25 Marks)

1. Find the level curve of the function $f(x,y) = 4x^2 + y^2$ at the point $\left(\frac{\sqrt{3}}{2}, 1\right)$.
2. State the Extreme Value Theorem for Functions of Two Variables.
3. Let $w = x^2y - xy^3$ where $x = \cos t$ and $y = e^t$. Find $\frac{dw}{dt}$.
4. Evaluate $\int_0^{\frac{1}{2}} \int_0^{\sqrt{1-x}} 2xy\, dydx$
5. Find Jacobian of the transformation T defined by $x = e^u \cos 2v$ and $y = e^u \sin 2v$.
6. Write the formula for finding the area of the surface $z = f(x,y)$ defined over a region R in the xy plane.
7. Write an equivalent integral in polar coordinates for the integral $\int_{-2}^{2} \int_0^{\sqrt{4-x^2}} e^{x^2+y^2}\, dydx$.
8. Write the relation between the rectangular coordinates (x, y, z) and the spherical coordinates $(\rho, \theta, \emptyset)$.
9. Define the line integral of a vector field $\vec{F}$ along a smooth curve C.
10. Define curl of a vector field $\vec{F}$.
11. Determine whether the vector field $\vec{F}(x,y) = 2xy^2\hat{\imath} + x^2y\hat{\jmath}$ is conservative or not.
12. Define a potential function for a vector field $\vec{F}$.
13. Write a parametric equation for the cone $x = \sqrt{y^2 + z^2}$.
14. State Stoke's Theorem.
15. Define a smooth surface.

SECTION B: Answer the following questions. Each carries *five* marks.
(Ceiling 35 Marks)

16. Find equations of the tangent plane and normal line to the surface with equation $xy + yz + xz = 11$ at the point $P(1,2,3)$.
17. Evaluate $\iint_R (x^2 + y)dA$ where R is the region bounded by the graphs of $y = x^2 + 2, x = 0, x = 1$ and $y = 0$.
18. Evaluate $\iiint_T zdV$ where T is the solid in the first octant bounded by the graph of $z = 1 - x^2$ and $y = x$.

(PTO)

19. Find the directional derivative of $f(x, y) = e^x \cos 2y$ at the point $\left(0.\frac{\pi}{4}\right)$ in the direction of $\vec{v} = 2\hat{\imath} + 3\hat{\jmath}$.

20. Evaluate $\int_C 2x\, ds$ where C consists of the arc C_1 of the parabola $y = x^2$ from $(0,0)$ to $(1,1)$ followed by the line segment C_2 from $(1,1)$ to $(0,0)$.

21. Find curl $\vec{F}$ if $\vec{F}(x, y, z) = xy\hat{\imath} + xz\hat{\jmath} + xyz^2\hat{k}$ and hence find curl$\vec{F}(-1,2,1)$.

22. Find the surface area of the part of the paraboloid $\vec{r}(u, v) = u \cos v\, \hat{\imath} + u \sin v\, \hat{\jmath} + u^2\hat{k}$ where $0 \leq u \leq 3$ and $0 \leq v \leq 2\pi$.

23. Evaluate $\iint_S x + y\, dS$ where S is the part of the plane $3x + 2y + z = 6$ in the first octant.

SECTION C: Answer any *two* questions. Each carries *ten* marks.

24. If $\vec{F}(x, y, z) = 2xy^2z^3\hat{\imath} + 2x^2yz^3\hat{\jmath} + 3x^2y^2z^2\hat{k}$,

(a) Show that $\vec{F}$ is conservative and find a function f such that $\vec{F} = \nabla f$.

(b) If $\vec{F}$is a force field, find the work done by $\vec{F}$in moving a particle along any path from $(0,0,0)$ to $(1,1,1)$.

25. State the Second Derivative Test for a function of two variables. Find and classify the relative extrema and saddle points of the function of $f(x, y) = 2x^2 + y^2 - 2xy - 8x - 2y + 2$.

26. Evaluate $\iint_R x + y\, dA$ where R is the parallelogram bounded by the lines with equations $y = -2x, y = \frac{1}{2}x - \frac{15}{2}, y = -2x + 10, y = \frac{1}{2}x$. T is defined by $x = u + 2v$ and $y = v - 2u$

27. (a) Using Green's theorem, evaluate $\oint_C (x^2y + x^3)\, dx + 2xy\, dy$ where C is the region bounded by $y = x^2$ and $y = x$.

(b) Let T be the region bounded by the parabolic cylinder $z = 1 - y^2$ and the planes $z = 0, x = 0$ and $x + z = 2$ and let S be the surface of T.

If $\vec{F}(x, y, z) = xy^2\hat{\imath} + \left(\frac{1}{3}y^3 - \cos xz\right)\hat{\jmath} + xe^y\hat{k}$, find $\iint_S \vec{F}.\vec{n}\, dS$.

(2 × 10 = 20 Marks)