

## FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2023

(Regular/Improvement/Supplementary)

## MATHEMATICS: COMPLEMENTARY COURSE FOR PHYSICS, CHEMISTRY &amp; C S

## GMAT4C04T: MATHEMATICS - 4

Time: 2 Hours

Maximum Marks: 60

SECTION A: Answer the following questions. Each carries *two* marks.

(Ceiling 20 Marks)

1. Show that the sequence  $\left\{\frac{\ln(n)}{n}\right\}$  converges to zero.
2. State the sandwich theorem of sequence.
3. Show that series  $\sum \frac{1-n}{1+2n}$  diverges.
4. Use the root test to check the convergence of  $\sum \frac{1}{n^n}$ .
5. Show that the series  $\sum_{n=0}^{\infty} \frac{x^n}{n!}$  converges absolutely for all  $x$ .
6. Find the Laplace transformation of  $\sin(3t)\cos(5t)$ .
7. Define the unit step function  $u(t-a)$ . Draw the graph of the function  $1 - u(t-a)$ .
8. Define the Dirac's Delta function and WRITE its Laplace transformation.
9. Define a periodic function. Find the fundamental period of  $\tan(\pi x)$ .
10. Write the One-dimensional Wave equation and One-dimensional Heat equation.
11. Verify whether the function  $u = e^x \sin y$  is a solution of the Two-dimensional Laplace's equation.
12. Write the formula in the classical Runge-Kutta method of fourth order while solving the ivp  $y' = f(x,y)$ ,  $y(x_0) = y_0$ .

SECTION B: Answer the following questions. Each carries *five* marks.

(Ceiling 30 Marks)

13. Drop a ball from a height 'a' meters above the surface. Each time the ball hits the surface after falling a distance  $h$ , it rebounds a distance  $rh$ , where  $r$  is positive but less than 1. Find the total distance travelled by the ball before it comes to rest.
14. Find the sum of the telescoping series  $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$
15. Test the convergence of the series  $\sum_{n=1}^{\infty} \frac{n^n}{n!}$ .
16. Find the Taylor series and Taylor polynomials generated by  $f(x) = \sin x$  at  $x=0$ .
17. State the Convolution theorem. Using it find the inverse Laplace transform of  $\frac{s}{(s+1)(s^2+9)}$
18. Find the Laplace transformation of  $f(t) = t \cdot \cosh t$ .
19. Use Simpson's rule with  $n = 4$  to approximate  $\int_0^1 5x^4 dx$ .

SECTION C: Answer any *one* question. Each carries *ten* marks.

20. Using Laplace transformation, solve the ivp  $y'' + y' - 6y = 1$ ,  $y(0) = 0$ ,  $y'(0) = 1$ .
21. Find the Fourier series of the function  $f(x) = x$ ,  $-\pi < x < \pi$  and  $f(x+2\pi) = f(x)$ .

(1 × 10 = 10 Marks)