FOURTH SEMESTER B. Sc. DEGREE EXAMINATION, APRIL 2023

(Regular/Improvement/Supplementary)

ECONOMICS & MATHEMATICS (DOUBLE MAIN)

GDMT4B05T: DISTRIBUTION THEORY

Time: 2 ½ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries *two* marks.

(Ceiling 25 Marks)

1. The joint p.m.f of a discrete bivariate random variable is given below.

X \ Y	1	2	3
1	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{2}{10}$
2	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{1}{10}$

Find the marginal distribution of Y

2. Let X be a random variable with p.d.f $f(x) = \begin{cases} \frac{1}{2}, & -1 < x < 1 \\ 0, & otherwise \end{cases}$

   Obtain m.g.f.

3. A continuous random variable X has the p.d.f $f(x) = 1$; $0 \leq x \leq 1$.
   Find the first raw moment μ_1'.

4. Write down the mean and variance of a binomial distribution B(10,0.5)

5. The mean and variance of a Binomial distribution are 4 and $\frac{4}{3}$ respectively.

   Find n and p.

6. For a Poisson distribution mean = 4. Find it's standard deviation.

7. State lack of memory property of Geometric distribution.

8. Define beta distribution of first kind.

9. Write down the mean and variance of a gamma distribution.

10. Let $X \sim N(\mu, \sigma^2)$. Write down the m.g.f.

11. If Z is a standard normal variate, find P(Z > 1).

12. Define exponential distribution.

13. Let a random variable $X \sim U(0,1)$. Find it's mean and variance.

14. Distinguish between parameter and statistic.

15. Define convergence in probability.

SECTION B: Answer the following questions. Each carries *five* marks.

(Ceiling 35 Marks)

16. Define probability density function. Examine whether the following is a probability density function

$$f(x) = 3x^2 \; ; \; 0 \le x \le 1$$

17. If the joint p.d.f of (X, Y) is given by f(x , y) = 2 - x - y , $0 < x < 1$, $0 < y < 1$.
    Find the marginal probability density functions of X and Y.

18. If X follows Poisson distribution such that P(X = 1) = P (X = 2), find mean. Also find P(X = 4).

19. The number of printing mistakes per page in a book follows a poisson distribution with parameter 1.5. Find the probability that (i) There is no printing mistake in a particular page (ii) There are exactly 2 printing mistakes in a particular page.

20. Let X follows a one parameter gamma distribution with parameter = 1. Find it's mean, variance and m.g.f.

21. Height of students is normally distributed with mean 165 cm and SD 5 cm. Find the probability that height of student is (i) More than 175 cms and (ii) Less than 168 cms.

22. State and prove weak law of large numbers.

23. Define the following.
    (i) Chi- square distribution (ii) t-distribution (iii) F distribution.

SECTION C: Answer any *two* questions. Each carries *ten* marks.

24. The p.m.f of discrete random variable is given below

x	-3	-2	-1	0	1	2	3
P(x)	0.5	0.1	0.05	0.05	0.2	0.04	0.06

    a) Find E(X)
    b) Find E(2X+3)
    c) Find E(4X+5)
    d) Find E(X²)

25. For a binomial distribution mean = 6 and variance = 2.4 Find.
    a) Moment generating function
    b) P(x=2)
    c) P(X>4)
    d) Probability generating function.

26. What do you mean by rectangular distribution? Write down it's p.d.f , mean, variance and m.g.f. If X follows a rectangular distribution U(-1,1) find (i) P(0<X<1)(ii)P(X>0.5) (iii) P(0-.5<X<0) (iv) P(X>0)

27. Let X be a random variable taking values $-$ 1, 0 , +1 with probabilities $\frac{1}{8}, \frac{6}{8}, \frac{1}{8}$ respectively.

    Using Chebyshev's inequality, find the upper bound of the probability $P[|X| \ge 1]$.

(2 x 10 = 20 Marks)