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**FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2023**

**(Regular/Improvement/Supplementary)**

**ECONOMICS & MATHEMATICS (DOUBLE MAIN)**

**GDMT4B04T: ABSTRACT ALGEBRA**

**Time: 2 ½ Hours**

**Maximum Marks: 80**

**SECTION A: Answer the following questions. Each carries two marks.**

**(Ceiling 25 Marks)**

1. Is every abelian group cyclic.
2. Find  $\Phi(10)$ , where  $\Phi$  is the Euler's function.
3. In  $Z_7$ , find  $([4]_7)^{-1}$
4. If  $\sigma = (1\ 4\ 2\ 3) \in S_4$ , find  $\sigma^{-1}$ .
5. Define an abelian group.
6. Is  $Z_n$  cyclic?
7. Define kernel of a group homomorphism.
8. Find the order of the permutation  $(1\ 2\ 4)$  in  $S_5$ .
9. List the elements of  $S_3$ .
10. Find all cyclic subgroups of  $Z_4$ .
11. In  $Z_5$ , find the order of 2.
12. Let  $F$  be a field. For all  $a \in F$ , prove that  $-(-a) = a$ .
13. Let  $G$  be Group, define an automorphism of  $G$ .
14. Is  $Z_{11}$  simple?
15. Is  $\langle \mathbb{R}, + \rangle \cong \langle \mathbb{R}^+, \cdot \rangle$ . Justify.

**SECTION B: Answer the following questions. Each carries five marks.**

**(Ceiling 35 Marks)**

16. State and prove Fundamental homomorphism theorem.
17. Let  $\Phi: G_1 \rightarrow G_2$  be a group homomorphism with kernel  $\Phi = K$   
Prove that the homomorphism is 1-1 if and only if  $K = \{e\}$ .
18. Give examples for even and odd permutations in  $S_5$ .
19. Find the order of the permutation  $(1\ 3\ 5\ 4)(3\ 1)$  in  $S_6$ .
20. Is  $Z_7$  an integral domain?
21. State and prove second isomorphism theorem.
22. Make the addition and multiplication table for  $Z_6$ .
23. Draw the subgroup diagram of  $Z_9$ .

**SECTION C: Answer any two questions. Each carries ten marks.**

24. State and prove Lagrange theorem.
25. In  $Z_{11}$ , a) Find the order of [2], [5], [6],[7],[9]  
b) Find the inverse of [3],[4],[8],[10].
26. State and prove Cayley 's theorem.
27. a) Consider  $S_7$ , express the following permutations as product of disjoint cycles and Product of transpositions. Also find their order.  
i)  $(1\ 4\ 5\ 7)(4\ 2\ 5)(2\ 3)$   
ii)  $(3\ 5\ 6)(2\ 5\ 6)(4\ 3\ 1)$
- b) Explain the nonisomorphic groups of order 4.

**(2 × 10 = 20 Marks)**