FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2023

(Regular/Improvement/Supplementary)

ECONOMICS & MATHEMATICS (DOUBLE MAIN)

GDMT4A02T: THEORY OF EQUATIONS AND COMPLEX NUMBERS

Time: 2 ½ Hours   Maximum Marks: 80

SECTION A: Answer the following questions. Each carries *two* marks.
(Ceiling 25 Marks)

1. By the method of detached coefficients, divide $x^5 - 3x^2 + 6x - 1$ by $x^2 + x + 1$.

2. Find the upper limit of the moduli of roots for the equation $2x^4 - 7x^3 + 6x^2 - 5 = 0$.

3. Divide $x^4 - x^2 + 5x - 6$ by $x + 2$ using synthetic division.

4. Expand $x^4 - 6x^2 + 1$ in powers of $x + 2$.

5. Find a lower limit of negative roots of the equation $2x^6 + 20x^5 + 30x^3 + 50x + 1 = 0$.

6. Give an example of a biquadratic equation.

7. State fundamental theorem of algebra.

8. Solve trigonometrically $y^3 - 3y + 1 = 0$.

9. Define symmetric function. Give an example.

10. The equation $x^3 - 3x - 3 = 0$ has at least one real root. State true/false. Justify.

11. State Descartes rule of signs.

12. Express $\frac{2-4i}{3+5i}$ in the $a + ib$ form.

13. Find the principal argument of $1 + i$.

14. Give the parametric representation of a circle with centre z_0 and radius r.

15. Find the complex conjugate of $z = 1 - 4i$.

SECTION B: Answer the following questions. Each carries *five* marks.
(Ceiling 35 Marks)

16. State and prove remainder theorem.

17. Find the highest common divisor of $x^6 + 2x^5 + x^3 + 3x^2 + 3x + 2$ and $x^4 + 4x^3 + 4x^2 - x - 2$.

18. Solve by Cardan's method $x^3 - 15x - 126 = 0$.

19. Find the rational roots of the equation $6x^4 - 7x^3 + 8x^2 - 7x + 2 = 0$.

20. Separate the roots of the equation $2x^5 - 5x^4 + 10x^2 - 10x + 1 = 0$.

21. Verify that $(x_1 + x_2 - x_3 - x_4)(x_1 + x_3 - x_2 - x_4)(x_1 + x_4 - x_2 - x_3)$ is symmetric and break it into sigma functions.

22. Explain the three special types of linear functions.

23. Find the modulus and argument of: (i) $z_1 = -i$ (ii) $z_2 = 1 + \sqrt{3}i$

SECTION C: Answer any *two* questions. Each carries *ten* marks.

24. (i) Find the sum of squares of roots of the equation $2x^4 - 8x^3 + 6x^2 - 3 = 0$.

    (ii) Solve the equation $x^5 - x^4 - 2x^3 + 2x^2 + x - 1 = 0$ given that the equation has multiple roots

25. Solve $x^4 - 8x^2 - 4x + 3 = 0$ using Ferrari's method.

26. Show that for all real values of λ, the roots of the equation.

    $f(x) = (x - 1)(x - 3)(x - 5)(x - 7) + \lambda(x - 2)(x - 4)(x - 6) = 0$ are simple and separate them.

27. (i) If $z = \frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$, show that $z^2 + i = 0$.

    (ii) Evaluate $\left(\frac{1}{2} + \frac{1}{2}i\right)^{10}$.

    (2 × 10 = 20 Marks)