| QP CODE: D3BMT2403 | (Pages: 3) | Reg. No | : ............................... |
| --- | --- | --- | --- |
| | | Name | : ............................... |

**THIRD SEMESTER FYUGP EXAMINATION, NOVEMBER 2025**

**Discipline Specific Core (DSC) Courses - Major**

**MAT3CJ202 : Matrix Algebra**

**(Credits: 4)**

| Time: 2 Hours | Maximum Marks: 70 |
| --- | --- |

## Section A

**Answer the following questions. Each carries 3 marks (Ceiling: 24 marks)**

| | | | |
| --- | --- | --- | --- |
| 1. | For $v_1,\ v_2,\ v_3$ in $\mathbb{R}^n$ , write the linear combination $3v_1 - 5v_2 + 7v_3$ as a matrix times a vector. | BL1 | CO1 |
| 2. | Give an example of a $3 \times 3$ matrix in row echelon form but not in reduced row echelon form and give explanation. | BL2 | CO1 |
| 3. | Find the general solution of the system whose augmented matrix is given: $$\begin{bmatrix} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{bmatrix}$$ | BL3 | CO1 |
| 4. | Define when a mapping $T : \mathbb{R}^n \to \mathbb{R}^m$ is said to be onto and one-to-one. | BL1 | CO3 |
| 5. | Find the standard matrix $A$ for the dilation transformation $T(x) = 3x$ for $x$ in $\mathbb{R}^2$. | BL2 | CO3 |
| 6. | Describe and compare the solution sets of $x_1 + 9x_2 - 4x_3 = 0$ and $x_1 + 9x_2 - 4x_3 = -2$. | BL3 | CO1 |
| 7. | Define invertible matrix and give one example. | BL1 | CO1, CO3 |
| 8. | The given vector $x$ is in a subspace $H$ with a basis $\mathcal{B} = \{b_1, b_2\}$. $$b_1 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}, \quad b_2 = \begin{bmatrix} -2 \\ 7 \end{bmatrix}, \quad x = \begin{bmatrix} -3 \\ 7 \end{bmatrix}$$ Find the $\mathcal{B}$-coordinate vector of $x$. | BL2 | CO1 |
| 9. | Define determinant of an $n \times n$ matrix $A$. | BL1 | CO2 |
| 10. | Is $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$ an eigenvector of $\begin{bmatrix} -3 & 1 \\ -3 & 8 \end{bmatrix}$ ? If so, find the eigenvalue. | BL2 | CO2 |

**(PTO)**

## Section B

**Answer the following questions. Each carries 6 marks (Ceiling: 36 Marks)**

| | | | |
|---|---|---|---|
| 11. | Solve the system: $$\begin{aligned} x_1 - 3x_2 &= 5 \\ -x_1 + x_2 + 5x_3 &= 2 \\ x_2 + x_3 &= 0 \end{aligned}$$ | BL2 | CO1 |
| 12. | Let $a_1 = \begin{bmatrix} 1 \\ -2 \\ -5 \end{bmatrix}$, $a_2 = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$ and $b = \begin{bmatrix} 7 \\ 4 \\ -3 \end{bmatrix}$. Determine whether $b$ can be generated (or written) as a linear combination of $a_1$ and $a_2$. That is, determine whether weights $x_1$ and $x_2$ exist such that $$x_1 a_1 + x_2 a_2 = b \qquad (1)$$ If the vector equation (1) has a solution, find it. | BL1 | CO1 |
| 13. | Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$. <br><br> a) Determine if the set $\{v_1, v_2, v_3\}$ is linearly independent. <br><br> b) If possible, find a linear dependence relation among $v_1, v_2, v_3$. | BL1 | CO1 |
| 14. | Let $A = \begin{bmatrix} 1 & -4 & 7 & -5 \\ 0 & 1 & -4 & 3 \\ 2 & -6 & 6 & -4 \end{bmatrix}$ and $b = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$. Is $b$ in the range of the linear transformation $x \longmapsto Ax$ ? Why or why not? | BL3 | CO1 |
| 15. | If $v_1$ and $v_2$ are in $\mathbb{R}^n$ and $H = Span\{v_1, v_2\}$ , then show that $H$ is a subspace of $\mathbb{R}^n$. | BL1 | CO3 |
| 16. | Let $\mathrm{u} = \begin{bmatrix} -2 \\ 3 \\ -4 \end{bmatrix}$ and $\mathrm{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$. Compute $\mathrm{u}^T\mathrm{v}$, $\mathrm{v}^T\mathrm{u}$, $\mathrm{u}\mathrm{v}^T$, and $\mathrm{v}\mathrm{u}^T$. | BL2 | CO1, CO3 |
| 17. | Let $A = \begin{bmatrix} -2 & -7 & -9 \\ 2 & 5 & 6 \\ 1 & 3 & 4 \end{bmatrix}$. Find the third column of $A^{-1}$ without computing the other columns. | BL3 | CO1 |
| 18. | Let $A = PDP^{-1}$ and compute $A^4$, where $P = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$ and $D = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$. | BL2 | CO1, CO2 |

| Section C | | | |
|---|---|---|---|
| **Answer any one question. Each carries 10 marks (1 x 10 = 10 Marks)** | | | |
| 19. | a) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the transformation that reflects each vector $\mathbf{x} = (x_1, x_2, x_3)$ through the plane $x_3 = 0$ onto $T(\mathbf{x}) = (x_1, x_2, -x_3)$ . Show that $T$ is a linear transformation.<br>b) The matrix $\begin{bmatrix} 4 & -2 & 5 & -5 \\ -9 & 7 & -8 & 0 \\ -6 & 4 & 5 & 3 \\ 5 & -3 & 8 & -4 \end{bmatrix}$ determines a linear transformation $T$. Find all $\mathbf{x}$ such that $T(\mathbf{x}) = \mathbf{0}$. | BL3 | CO3 |
| 20. | Find the characteristic polynomial of each matrix;<br>a) $\begin{bmatrix} 0 & 3 & 1 \\ 3 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$<br>b) $\begin{bmatrix} 6 & -2 & 0 \\ -2 & 9 & 0 \\ 5 & 8 & 3 \end{bmatrix}$ | BL2 | CO2 |
| | | | |
| | **CO : Course Outcome** | | |
| | **BL : Bloom's Taxonomy Levels** (1 – Remember, 2 – Understand, 3 – Apply, 4 – Analyse, 5 – Evaluate, 6 – Create) | | |