QP CODE: D3BAM2401	(Pages: 2)	Reg. No : .....
		Name : .....

THIRD SEMESTER FYUGP EXAMINATION, NOVEMBER 2025

Discipline Specific Core (DSC) Courses - Major

AMA3CJ201 : Vector Calculus

(Credits: 4)

Time: 2 Hours **Maximum Marks: 70**

Section A

Answer the following questions. Each carries 3 marks (Ceiling: 24 marks)

1.	Determine the standard form equations for the tangent plane to the surface $z = \sqrt{x^2 + y^2}$ at $P_0(3, 1, \sqrt{10})$.	BL2	CO5
2.	Find div $\mathbf{F}$, given that $\mathbf{F} = \nabla f$ where $f(x, y, z) = xy^3z^2$	BL3	CO2, CO5
3.	Evaluate $\iint_R x^5y^2 dA$, where R is the rectangle $1 \leq x \leq 2,\ 0 \leq y \leq 1$ using an iterated integral with $x-$ integration first.	BL2	CO4
4.	Determine whether the vector field $\mathbf{F} = ye^{xy}\hat{i} + (xe^{xy} + x)\hat{j}$ is conservative; if it is, find a scalar potential.	BL1	CO4
5.	Evaluate $\int_1^2 \int_0^1 \int_{-1}^2 8x^2yz^3 dx\ dy\ \ dz$	BL2	CO4
6.	Let $z = x^2 sin(3x + y^3)$. Evaluate i) $\frac{\partial z}{\partial x}\|_{(\frac{\pi}{3},0)}$ ii) $\frac{\partial z}{\partial y}\|_{(1,1)}$	BL1	CO2
7.	State the "Second partials test".	BL1	CO2, CO3
8.	Use an appropriate volume formula to evaluate the double integral $\iint_R (4 - y)\ dA$ where $R : 0 \leq x \leq 3,\ 0 \leq y \leq 4$	BL2	CO4
9.	Define surface integral.	BL1	CO4
10.	Find the volume of the solid bounded above by the plane $z = y$ and below in the $xy-$plane by the part of the disk $x^2 + y^2 \leq 2$ in the first quadrant.	BL2	CO4

Section B

Answer the following questions. Each carries 6 marks (Ceiling: 36 Marks)

11.	Let f be the function defined by $f(x, y) = \begin{cases} \frac{xy^2}{x^2+y^4} & \text{for } (x, y) \neq (0, 0) \\ 0 & \text{for } (x, y) = (0, 0) \end{cases}$. Is f continuous at $(0, 0)$? Explain.	BL2	CO1, CO2
12.	Use the method of Lagrange multipliers to find the required constrained extrema: Maximize $f(x, y) = xy$ subject to $2x + 2y = 5$.	BL2	CO3

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13.	Suppose a region of $\mathbb{R}^3$ is heated so that its temperature T at each point (x, y, z) is given by $T(x, y, z) = 100 - x^2 - y^2 - z^2$ degree Celsius. Describe the isothermal surface for $T > 0$.	BL1	CO1, CO3
14.	Suppose f is a scalar function with continuous first and second partial derivatives exist in the simply connected region D and a piecewise smooth closed curve C bounds D. Show that $\iint_D \nabla^2 f \, dx \, dy = \oint_C \frac{\partial f}{\partial n} ds$ where $\nabla^2 f = f_{xx} + f_{yy}$ is the Laplacian of f and $\frac{\partial f}{\partial n} = \nabla f \cdot \mathbf{N}$ is the normal derivative vector.	BL1	CO4
15.	Evaluate $\int_0^3 \int_0^{\sqrt{9-x^2}} x \, dy \, dx$ by converting to polar coordinates.	BL2	CO4
16.	i) A right circular cylinder is changing in such a way that its radius r is increasing at the rate of $3 \, in/min$ and its height h is decreasing at the rate of $5 \, in/min$. At what rate is the volume of the cylinder changing when the radius is $10 \, in$ and height is $8 \, in$? ii) If y is a differantiable function of x such that $\sin(x + y) + \cos(x - y) = y$. Find $\frac{dy}{dx}$ using partial derivatives.	BL2	CO2
17.	Evaluate $\iint_S (curl \, \mathbf{F} \cdot N) \, dS$, where $\mathbf{F} = x\hat{i} + y^2\hat{j} + ze^{xy}\hat{k}$ and S is that part of the surface $z = 1 - x^2 - 2y^2$ with $z \geq 0$.	BL1	CO4
18.	Sketch the region of integration and then compute the integral $\int_0^1 \int_{-x^2}^{x^2} dy \, dx$ i) with the given order of integration. ii) with the order of integration reveresed.	BL2	CO4
	Section C		
	Answer any one question. Each carries 10 marks (1 x 10 = 10 Marks)		
19.	Let $\mathbf{F} = xy^2\hat{i} + x^2y\hat{j}$ be a vector field. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{R}$ between the points (0,0) and (2,4) along the following paths. i) the line segment connecting the points. ii) the parabolic curve $y = x^2$ connecting the points.	BL1	CO4
20.	i) In what direction is the function defined by $f(x, y) = xe^{2y-x}$ increasing most rapidly at the point $P_0(2, 1)$ and what is the maximum rate of increase? In what direction is f decreasing most rapidly? ii) Let $f(x, y, z) = xy \sin(xz)$. Find ∇f_0 at the point $P_0(1, -2, \pi)$ and then compute the directional derivative of f at P_0 in the direction of the vector $\mathbf{v} = -2\hat{i} + 3\hat{j} - 5\hat{k}$.	BL1	CO5
	CO : Course Outcome		
	BL : Bloom's Taxonomy Levels (1 – Remember, 2 – Understand, 3 – Apply, 4 – Analyse, 5 – Evaluate, 6 – Create)		