D3BST2301 (PAGES: 2) **Reg. No.............................**

Name: ....

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2024
(Regular/Improvement/Supplementary)
STATISTICS: Complementary Course for Mathematics & CS

GSTA3C03T: PROBABLITY DISTRUBUTION AND SAMPLING THEORY

Time: 2 Hours **Maximum Marks: 60**

SECTION A: Answer the following questions. Each carries *two* marks.
(Ceiling 20 marks)

1. Define chi-square statistic.
2. If X follows Normal with mean 40 and variance 4. Find
   (i) P(X < 50). (ii) P(X > 40).
3. Establish the relation connecting Bernoulli and Binomial random variables.
4. Define convergence in probability and convergence in distribution.
5. What is sampling?
6. Define stratified random sampling.
7. If X_1 , X_2 and X_3 are independent observations from N (0, 1), obtain the sampling distribution of $\dfrac{\sqrt{2}\,X_3}{\sqrt{X_1^2+X_2^2}}$.
8. Obtain the standard error of mean of a random sample of size n from a normal population with parameters μ and $\boldsymbol{\sigma}^2$.
9. Define degrees of freedom.
10. State Weak law of large numbers.
11. Define population.
12. Obtain the relation between t distribution and Cauchy distribution.

SECTION B: Answer the following questions. Each carries *five* marks.
(Ceiling 30 marks)

13. Prove that square of a Student's t random variable is F random variable.
14. $X \to N(\mu, \sigma)$. Prove that the $(2k)^{th}$ central moment $\mu_{2k} = (2k-1)\sigma^{2r}\mu_{2(k-1)}$ *for* k= 1, 2 , 3 .....
15. Explain systematic sampling.

(PTO)

16. (X_k), k = 1, 2, 3, … is a sequence of independent random variables each taking the value -1, 0, 1. Given that $P(X_k = 1) = P(X_k = -1) = \frac{1}{k}$ and $P(X_k = 0) = 1 - \frac{2}{k}$. Examine whether the law of large numbers holds for this sequence.

17. Examine whether WLLN holds for the sequence Xn of independent random variables.

$$P(X_n = \frac{1}{\sqrt{n}}) = \frac{2}{3} \quad , \quad P(X_n = \frac{-1}{\sqrt{n}}) = \frac{1}{3}.$$

18. If X_1 , X_2 , X_3 , . . . X_n is a random sample from a normal population with mean μ and variance $\boldsymbol{\sigma}^2$, obtain the distribution of $\frac{1}{n}\sum_{i=1}^{n} X_i$.

19. If $X \to P(\lambda)$. Show that $\mu_{r+1} = \lambda\left[r\,\mu_{r-1} + \frac{d\mu_r}{d\lambda} \right]$.

SECTION C: Answer any *one* question. The question carries *ten* marks.

20. For a continuous uniform distribution, $f(x) = \frac{1}{2a}; -a < X < a$. Show that

$\mu_{2r} = \frac{a^{2r}}{2r+1}$. Hence obtain its variance.

21. (i) State Chebyshev's inequality.

(ii) A random variable X has the values – 1, 1, 3 and 5 with probabilities $\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{2}$ respectively. Find, using Chebyshev's inequality, determine $P[|X - 3| \geq 3]$.

(1 x 10 = 10 Marks)