THIRD SEMESTER B. Sc. DEGREE EXAMINATION, NOVEMBER 2024

(Regular/Improvement/Supplementary)

MATHEMATICS: Complementary Course for Physics, Chemistry and C.S.

GMAT3C03T: MATHEMATICS - 3

Time: 2 Hours   **Maximum Marks: 60**

SECTION A: Answer the following questions. Each carries 2 marks.

(Ceiling 20 Marks)

1. Define line integral of a vector function $F(\tau)$ over a curve C.

2. Find a general solution of the differential equation $y'' - 6y' - 7y = 0$.

3. State Green's theorem in the plane.

4. Verify whether $y = 2e^{3t}$ is a solution of the differential equation $y' = 3y$. Justify.

5. Show that the integral $\int_C F.d\tau = \int_C (2xdx + 2ydy + 4zdz)$ is path independent in any domain in space.

6. Show that $\sin x$ and $\cos x$ are solutions of the ordinary differential equation $y'' + y = 0$.

7. Evaluate the integral $I = \int_C (3x^2 dx + 2yzdy + y^2 dz)$ from $A : (0,0,0)$ to $B : (2,2,2)$.

8. Define Curl of the vector function v.

9. State Stoke's theorem.

10. Find $a.b + b.c + c.a$ if $a = [2,1,4]$, $[b = [-4,0,3]$ and $c = [3,-2,1]$.

11. Find ∇f if $f = (x-3)^2 + (y-1)^2$.

12. Verify whether the differential equation $x^3 dx + y^3 dy = 0$ is exact. Justify.

SECTION B: Answer the following questions. Each carries 5 marks.

(Ceiling 30 Marks)

13. Find a general solution for the Euler-Cauchy equation $x^2 y'' - 5xy' + 9y = 0$.

14. Solve the differential equation $2xyy' = y^2 - x^2$.

15. Find the tangent to the ellipse $\frac{1}{4}x^2 + y^2 = 1$ at $P : (\sqrt{2}, \frac{1}{\sqrt{2}})$.

                                          (PTO)

16. Evaluate $\iint_S (7xi - zk).ndA$ over the sphere $S: x^2 + y^2 + z^2 = 4$.

17. Find the parametric representation of a cylinder $x^2 + y^2 = a^2$, $-1 \leq z \leq 1$.

18. Find the directional derivative of $f = xyz$ at $P = (-1, 1, 3)$ in the direction of $a = [1, -2, 2]$.

19. Find an integrating factor and solve the initial value problem
$(e^{x+y} + ye^y)dx + (xe^y - 1)dy = 0$, $y(0) = -1$.

SECTION C: Answer any 1 question. Each carries 10 marks.

20. Using method of undetermined coefficients, find the general solution of $y'' + 3y' + 2y = 30e^{2x}$.

21.   a) Find a normal vector of the surface $x^2 + 3y^2 + z^2 = 29$ at $P = (4, 1, 3)$.

   b) Find the divergence of the vector function $[x^3 + y^3, 3xy^2, 3zy^2]$.

(1 x 10 = 10 Marks)