D3BMC2301 **Reg. No.............................**

Name: ....

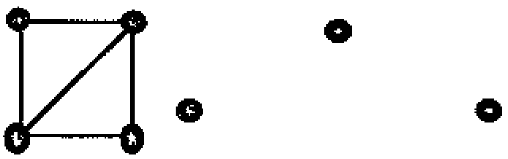
THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2024
(Regular/Improvement/Supplementary)
COMPUTER SCIENCE & MATHEMATICS (DOUBLE MAIN)
GDMA3A01T: BASIC LOGIC, BOOLEAN ALGEBRA AND GRAPH THEORY

Time: 2 ½ Hours **Maximum Marks: 80**

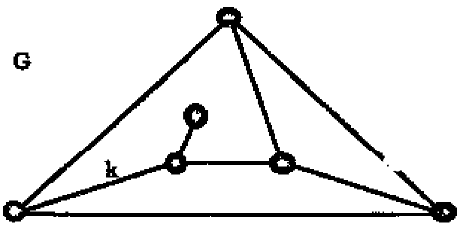
SECTION A: Answer the following questions. Each carries *two* marks.

(Ceiling 25 marks)

1. Let $p: \Delta ABC\ is\ equilateral$ and $q: \Delta ABC\ is\ isosceles$. Find $p \rightarrow q$ and $q \rightarrow p$.
2. When do we say that two lattices are isomorphic?
3. Is the set Z of integers with the usual order ≤ well-ordered? Justify.
4. Draw a complete graph with 6 edges.
5. Give an example for a self- complementary graph.
6. Find the number of components of the following graph.



7. What is meant by the connectivity of a graph?
8. Simplify the Boolean expression $p \vee (p \vee q)$.
9. Define a plane graph.Give one example.
10. Find the number of faces of the given graph.



11. State Dirac's theorem.
12. What do you mean by trivial proof?
13. Define a quasi-order on a set.
14. Is the set N of natural numbers ordered by divisibility, a totally ordered set? Justify.
15. What is meant by graph isomorphism?

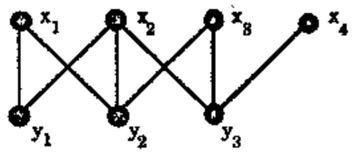
SECTION B: Answer the following questions. Each carries *five* marks.

(Ceiling 35 marks)

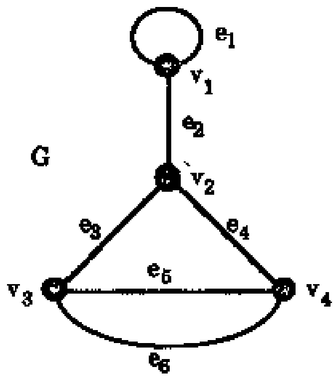
16. Prove by contradiction: There is no largest prime number; that is, there are infinitely many prime numbers.
17. Let G be a simple graph with n vertices and let u and v be non-adjacent vertices in G such that $d(u) + d(v) \geq n$. Let $G + uv$ denote the supergraph of Gobtained by joining u and v by an edge. Prove that G is Hamiltonian iff $G + uv$ is Hamiltonian.

(PTO)

18. For any positive integer m, we will let D_m denote the set of divisors of m ordered by divisibility. Draw the Hasse diagram of D_{36} $=\{1,2,3,4,6,9,12,18,36\}$.
19. Show by an example that a lattice can have join irreducible elements other than atoms.
20. Determine whether the following graph is bipartite and justify your claim.



21. Write the adjacency matrix of the following graph.



22. Give an example of a simple connected graph G with n vertices having a cut vertex v such that $\omega(G-v)=n-1$ and each connected component of $G-v$ consists of an isolated vertex.
23. Define converse, inverse and contrapositive. Give converse, inverse and contrapositive of the given implication
    $p \to q: If\ \Delta ABC\ is\ equilateral, then\ it\ is\ isosceles.$

SECTION C: Answer any *two* questions. Each carries *ten* marks.

24. Using the laws of logic simplify the Boolean expression $(p\wedge \sim q)\vee q\vee(\sim p\wedge q)$.
25. Let L be a complemented lattice with unique complements. Then prove that the join irreducible elements of L, other than 0, are its atoms.
26. Prove that an edge e of a graph G is a bridge if and only if e is not part of any cycle in G
27. a) State Dirac Theorem.
    b) Define closure of a graph.
    c) A simple graph G is Hamiltonian if and only if its closure C(G) is Hamiltonion.

(2 x 10 = 20 Marks)