D3BHM2303 (PAGES: 3) **Reg. No.............................**

Name: ....

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2024
(Regular/Improvement/Supplementary)
HONOURS IN MATHEMATICS
GMAH3B12T: DIFFERENTIAL EQUATIONS

Time: 3 Hours **Maximum Marks: 80**

Part A. Answer *all* the questions. Each carries *one* mark.
Choose the correct answer.

1. The ordinary differential equation $\frac{2}{3}\frac{dy}{dx} = y^{1/3}$, $y(0) = 0$ has _________.

   A) Infinite many solutions. B) Unique solution.

   C) No solution. D) Exactly two solution.

2. The characteristic equation of the differential equation $y'' - 4y' + 4y = 0$ is:

   A) $(\lambda - 2)^3 = 0$. B) $(\lambda + 2)^2 = 0$.

   C) $(\lambda - 2)^2 = 0$. D) $(\lambda - 1)(\lambda - 2) = 0$.

3. The Laplace transform of the unit step function $u_a(t)$ is:

   A) e^{-as} B) $\frac{e^{-as}}{s}$ C) $\frac{e^{as}}{s}$ D) $\frac{e^{-as}}{s^2}$

4. If $\mathcal{L}\{f(t)\} = F(s)$, then $\mathcal{L}\{e^{at}f(t)\} =$

   A) $F(s)$ B) $F(s-a)$ C) $F(s+a)$ D) $F(\frac{s}{a})$

5. Which of the following is a boundary value problem?

   A) $y'' + y = 0$, $y(0) = 1$, $y'(0) = 0$
   B) $y'' + 7y = 0$, $y(0) = 1$, $y'(0) = 1$
   C) $y'' + y = 0$, $y(0) = 1$, $y(1) = 0$
   D) $y''' + y'' + y = 0$, $y(0) = y'(0) = y''(0) = 0$

Fill in the Blanks.

6. The general solution of $xy\frac{dy}{dx} - 1 = 0$ is ____.
7. A particular solution of the differential equation $y'' + y = \tan x$ is ________________.
8. If $\delta(t-1)$ is Dirac delta function, then the $\mathcal{L}\{\cos 3t\,\delta(t-1)\}$ = ___________________.
9. The Laplace transform of the function $f(t) = \begin{cases} e^t, & 0 < t < 1 \\ 0, & t > 1 \end{cases}$ is ___________________.
10. The Eigen function of the boundary value problem $y'' + \lambda y = 0$, $y(0) = y(L) = 0$ is _________________.

(10 x 1 = 10 Marks)

(PTO)

Part B. Answer any *eight* questions. Each carries *two* marks.

11. Solve $xdy - ydx = xy^2dx$.
12. What is the difference between particular and singular solutions of ordinary differential equations?
13. Find the general solution of the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 7y = 0$.
14. The Wronskian of two functions is $W(t) = t\,sin^2t$. Are the functions linearly independent or dependent? Why?
15. If $y_1 = 1$ is one solution of the equation $xy'' + 3y' = 0$, then find another solution y_2 of this equation.
16. Find the Laplace transform of $f(t) = t^2 - 3t + 5$.
17. Using definition of Laplace transform, prove that $\mathcal{L}\{\mathrm{e}^{\mathrm{at}}\} = \frac{1}{\mathrm{s-a}}$ for $s > a$.
18. Evaluate $\mathcal{L}^{-1}\left\{\frac{3\mathrm{s}-8}{\mathrm{s}^2-16}\right\}$.
19. Solve the differential equation $y' - y = 0,\quad y(0) = 1$, by using Laplace transform.
20. If $\mathrm{f(x)} = |\mathrm{x}|,\ -\pi < x < \pi$, then find a_0 in the Fourier series expansion of f(x).

(8 x 2 = 16 Marks)

Part C. Answer any *six* questions. Each carries *four* marks.

21. The differential equation $x\frac{dy}{dx} + 4y = 8x^4$ subjected to $y = 1$ at $x = 1$. Show that the solution of this differential equation is $y = x^4$.
22. Solve $(e^y + 1)\cos x\,dx + e^y \sin x\,dy = 0$.
23. Solve the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2e^x$.
24. Find a homogenous linear ODE for which two functions x^3 and x^{-2} are solutions. Also show their linear independence by considering their Wronskian.
25. State convolution theorem and use it to find inverse Laplace transform of $\frac{1}{s(s^2+1)}$.
26. If $\mathcal{L}\{f''(t)\} = tan^{-1}\left(\frac{1}{s}\right)$ given $f(0) = 3,\ f'(0) = -2$, then find $\mathcal{L}\{f(t)\}$.
27. Expand $f(x) = \pi x - x^2$ as a Fourier sine series in the interval $(0, \pi)$.
28. Find the solution to the heat conduction problem $\alpha^2 u_{xx} = u_{tt},\ 0 < x < 30,\ t > 0$

$$u(x,0) = 10, \qquad 0 < x < 30$$

$$u(0,t) = 0, \quad u(30,t) = 0, \quad t > 0$$

(6 x 4 = 24 Marks)

Part D. Answer any *two* questions. Each carries *fifteen* marks.

29. a) The velocity of a particle $\boldsymbol{v}$ at time **t** satisfies the differential equation $t\frac{dv}{dt} = v + t$, $t > 0$. Given that when $t = 2,\ v = 8$. Show that when $t = 8,\quad v = 16(2 + ln2)$.

    b) Find the solution of the initial value problem $y' = 1 - y^3,\quad y(0) = 0$ using the method of successive approximations.

30. a) Explain the method of undetermined co-efficient to solve non-homogenous linear differential equations with constant co-efficient.

    b) Obtain the general solution of the differential equation $y'' - 3y' + 2y = \frac{1}{1+e^{-x}}$ by using method of reduction of order.

31. a) Find the Fourier series expansion of $f(x) = \pi^2 - x^2$ in the interval $[-\pi, \pi]$. Hence deduce $\frac{\pi^2}{12} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} +$ ______.

    b) Consider an elastic string of length 60cm. Assume that the ends of the string are fixed and that the string is set in motion with no initial velocity from the initial position $u(x,0) = 60x - x^2,\ 0 < x < 60$. Find the displacement $u(x,t)$ of the string.

(2 x 15 = 30 Marks)