D3BHM2302 **(3 Pages)** **Reg.No:..................**
Name:....................

THIRD SEMESTER B. Sc. DEGREE EXAMINATION, NOVEMBER 2024
(Regular/Improvement/Supplementary)
HONOURS IN MATHEMATICS
GMAH3B11T: CALCULUS III

Time: 3 Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries 1 mark.

Choose the correct answer

1. In $\mathbb{R}^3$, the equation $y = x^2$ represents
   A) a parabola B) a cylinder C) a parabolloid D) a hyperboloid

2. What is the distance between the point $P(2, 3, -1)$ and the plane given by the equation $3x - 4y + 12z + 6 = 0$?
   A) $9/5$ B) $7/\sqrt{21}$ C) $15/\sqrt{29}$ D) $12/\sqrt{14}$

3. Which of the following describes the reflective property of an ellipse?
   A) The sum of distances from any point on the ellipse to the foci is constant.
   B) The difference of distances from any point on the ellipse to the foci is constant
   C) Light or sound waves emanating from one focus will pass through the other focus.
   D) The product of distances from any point on the ellipse to the foci is constant.

4. Consider the function $f(x, y) = \sqrt{25 - x^2 - y^2}$. What is the domain of this function?
   A) $x^2 + y^2 \leq 25$ B) $x^2 + y^2 > 25$ C) $x^2 + y^2 = 25$ D) $x^2 + y^2 \geq 25$

5. If the function $f(x, y) = x^2 - y^2 x^2 + y^2$ is evaluated at (x, y) close to the origin along different paths, which of the following is true?
   A) The limit exists and is the same for all paths.
   B) The limit does not exist because the function takes different values along different paths.
   C) The limit exists and is zero.
   D) The function is undefined for all paths leading to the origin.

Fill in the blanks

6. The standard symmetric equations of a line is ......

7. The domain of the vector function $\mathbf{r}(t) = \ln t\mathbf{i} + \cosh t\mathbf{j} + \tanh t$ is ......

8. If $\mathbf{r}(t) = t^2\mathbf{i} + \cos t\mathbf{j} + \mathbf{k}$, then $r'(t) = ...$

9. If $f(x, y) = x \cos xy^2$, then $\dfrac{\partial f}{\partial x} = ...$

10. The gradient of $f(x, y) = x \sin y + y \cos x$ at the point $(\pi/4, \pi/2)$ is ......

(10 x 1 = 10 Marks)

(PTO)

SECTION B: Answer any 8 questions. Each carries 2 marks.

11. Find parametric equations for the line passing through the point $P(-2,1,3)$ and parallel to the vector $\mathbf{v} =< 1, 2, -2 >$

12. Determine whether the planes $x - y + 2z = 5$ and $-3x + 3y - 6z = 11$ are parallel or not?

13. The point $(5, \pi/4, 3\pi/4)$ is expressed in spherical coordinates.
Find its cylindrical coordinates.

14. Define vector valued function.

15. Find the interval(s) on which the vector function defined by
$\mathbf{r}(t) = \dfrac{\cos t - 1}{t}\mathbf{i} + \dfrac{\sqrt{t}}{1+2t}\mathbf{j} - te^{-1/t}\mathbf{k}$ is continuous.

16. Find $\mathbf{r}''(t)$ if $\mathbf{r}(t) = \sqrt{t}\mathbf{i} + \dfrac{1}{t}\mathbf{j} + \ln t\mathbf{k}$.

17. Evaluate the integral $\displaystyle\int (\sin 2t\mathbf{i} + \cos 2t\mathbf{j} + e^{-t}\mathbf{k})dt$.

18. Find the domain and range of $g(x,y,z) = \sqrt{25 - x^2 - y^2 - z^2}$.

19. Define level curve.

20. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ if $x^3 + xy - x^2z + yz^2 = 0$.

(8 x 2 = 16 Marks)

SECTION C: Answer any 6 questions. Each carries 4 marks.

21. Show that the lines $L_1 : x = 1-t, y = -2-3t, z = 4$ and $L_2 : x = 2-2t, y = -4+3t, z = 1+4t$ are skew lines.

22. Sketch the curve defined by the vector function $\mathbf{r}(t) = 2t\mathbf{i} + (3t+1)\mathbf{j}$, $-1 \le t \le 2$.

23. Let $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$, where f, g and h are differentiable functions of t.
Then prove that $\mathbf{r}'(t) = f'(t)\mathbf{i} + g'(t)\mathbf{j} + h'(t)\mathbf{k}$.

24. Find the unit normal and unit tangent vectors for the curve defined by $\mathbf{r}(t) = \langle \sin 2t, \cos 2t, 3t \rangle$.

25. Sketch a contour map of the hyperbolic paraboloid defined by $f(x,y) = y^2 - x^2$.

26. Find all the second order partial derivatives of $f(x,y) = xe^{2y} + ye^{2x}$

27. Find the directional derivative of $f(x,y,z) = x^2y\cos 2z$ at the point $(-1, 2, \pi/4)$ in the direction of $\mathbf{v} = \mathbf{i} - \mathbf{j} + \mathbf{k}$.

28. If $z = f(x,y)$ where $x = r\cos\theta$ and $y = r\sin\theta$, show that $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$

(6 x 4 = 24 Marks)

SECTION D: Answer any 2 questions. Each carries 15 marks.

29. Find parametric equations for the line of intersection of the planes $3x + y - 2z = 4$ and $2x - y - 3z = 6$. Also find the angle between the two planes.

30. (a) Find the curvature of the curve $y = e^{-x^2}$.

    (b) Find the point(s) where the curvature is zero.

31. If $z = f(x, y)$ where $x = r\cos\theta$ and $y = r\sin\theta$ show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 z}{\partial \theta^2} + \frac{1}{r}\frac{\partial z}{\partial r}$$

(2 x 15 = 30 Marks)