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D3BHM2301 **Reg. No.............................**

Name: ....

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2024

(Regular/Improvement/Supplementary)

HONOURS IN MATHEMATICS

GMAH3B10T: REAL ANALYSIS - I

Time: 3 Hours **Maximum Marks: 80**

Part A. Answer *all* the questions. Each carries *one* mark.

Choose the correct answer.

1. Number of elements in $\mathcal{P}(S)$, where S= {1,2} is:

   A) 2 B) 4 C) 6 D) 8

2. The set of all $x \in \boldsymbol{R}$ that satisfy the inequalities $|x^2 - 1| \leq 3$ is

   A) $-2 \leq x \leq 2$ B) $-1 \leq x \leq 2$ C) $-2 \leq x \leq 1$ D) $-1 \leq x \leq 1$

3. Let S = $\{1 - \frac{(-1)^n}{n}, n \in N\}$. Find sup S.

   A) 1 B) 0 C) 2 D) $\frac{1}{2}$

4. If $I_n = \left[0, \frac{1}{n}\right], n \in \boldsymbol{N}$, then $\bigcap_{n=1}^{\infty} I_n =$ ___________.

   A) $\emptyset$ B) {0} C) {1} D) [0,1]

5. $\lim_{n \to \infty} \frac{3n+1}{n+2} =$ _____________.

   A) 1 B) 2 C) 3 D)0

Fill in the Blanks.

6. If a ϵ R is such that $0 \leq a < \varepsilon$ for every $\varepsilon > 0$, then a = ___________.
7. Every nonempty set of real numbers that has an upper bound also has a ___________ in R.
8. The set (a,a) = __________, for any real number a.
9. The rational expression for 7.31414141... is ___________.
10. __________ is an example of a bounded sequence that is not Cauchy.

(10 × 1 = 10 Marks)

Part B. Answer any *eight* questions. Each carries *two* marks.

11. Define positive real numbers. Show that $1 > 0$.
12. If $A_i = \{i, i+1, i+2, ...\}$ find (i) $\bigcup_{i=1}^{n} A_i$ and (ii) $\bigcap_{i=1}^{n} A_i$.
13. State and Prove Bernoulli's inequality.
14. Prove that $\big||a| - |b|\big| \leq |a - b|$ ∀ a, b ∈ ℝ.
15. Show that if A and B are bounded subsets of ℝ then A∪B is a bounded set. Show that sup A∪B = sup {sup A, sup B}.
16. Using Archimedean property show that inf $\{\frac{1}{n} : n \in \mathbb{N}\} = 0$.

(PTO)

17. Show that a sequence of real numbers can have utmost one limit.
18. Prove that every Cauchy sequence is bounded.
19. Show by a suitable example that intersection of infinitely many open sets in ℝ need not be open.
20. Show that the set N of natural numbers is a closed set in R.

(8 × 2 = 16 Marks)

Part C. Answer any *six* questions. Each carries *four* marks.

21. Show that set $\boldsymbol{Q}$, of rationals is countable.
22. If x and y are real numbers with x < y, then prove that ∃ an irrational number z such that x < z < y.
23. Establish that every monotone sequence is convergent if and only if it is bounded.
24. If $x_1 = 2, x_{n+1} = 2 + \frac{1}{x_n}, \forall\, n \geq 2$, find lim x_n.
25. Let (x_n) be a bounded sequence and let s = sup$\{x_n : n \in \mathbb{N}\}$. Show that if s $\neq x_n$ for any n, there is a subsequence of (x_n) that converges to s.
26. If $F \subseteq \boldsymbol{R}$ prove that F is closed if and only if it contains all its cluster points.
27. State and prove nested interval property.
28. Show that the Cantor set contains no nonempty open interval as a subset.

(6 × 4 = 24 Marks)

Part D. Answer any two questions. Each carries *fifteen* marks.

29. Let S and T be sets and T ⊆ S. Prove the following:
    (a) If S is countable, then T is countable.
    (b) If T is uncountable then S is uncountable.
30. Show that there exists a sequence of real numbers (S_n) that converges to $\sqrt{2}$.
31. (a) If lim $x_n = x$ and lim $y_n = y$, prove that lim $\frac{x_n}{y_n} = \frac{x}{y}$.

    (b) Discuss the convergence of the sequence x_n where:

    (i) $x_n = \frac{(-1)^n n}{n^2+1}$

    (ii) $y_n = \frac{a^{n+1}+b^{n+1}}{a^n+b^n}$, where 0 < a < b

(2 × 15 = 30 Marks)