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D3BEM2305 **Reg. No.............................**

Name: ....

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2024
(Regular/Improvement/Supplementary)
ECONOMICS & MATHEMATICS (DOUBLE MAIN)
GDMT3B03T: MULTIVARIABLE CALCULUS

Time: 2 ½ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries *two* marks.
(Ceiling 25 marks)

1. Define the Jacobian used to transform 3 variable functions.
2. Define line integral of a function $f(x, y)$ along a curve C .
3. Find the gradient of the function $f(x, y) = xsiny$ at the point (e, π).
4. Find the value of the double integral by $\iint_R 2\, dA$ where $R = [-1,3]\ x\ [2,5]$ interpreting it as the volume of a solid.
5. State Fubini's theorem for rectangular regions.
6. What are conservative vector field and potential function?
7. Transform the double integral $\iint_R y\, dA$ where R is the region in the first quadrant that is outside the circle $r = 2$ and inside the cardioid $r = 2(1 + cos\theta)$ into polar coordinartes.
8. Define level curve of a function of 2 variables and give an example.
9. Write the equations to transform rectangular coordinates to cylindrical coordinates.
10. Write the formula to find the area of the surface $z = f(x, y)$.
11. Find the gradient vector field of the function $f(x, y, z) = x^2 + y^2 + z$.
12. Define curl of a function F(x,y,z).
13. Find the domain and range of the function $f(x, y) = x^2 + y^2$.
14. If $F(x, y) = P(x, y)i + Q(x, y)j$ is a conservative vector field in an open region R and both P and Q have continuous first order partial derivatives, then prove that $\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$ at each point (x, y) in R.
15. What is a saddle point?

(PTO)

SECTION B: Answer the following questions. Each carries *five* marks.
(Ceiling 35 marks)

16. Evaluate the double integral $\iint_R (2x + 3y)\, dA$ where R is the region in the first quadrant bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ by transforming into polar coordinates.
17. Prove that $\lim_{(x,y)\to(a,b)} y = b$.
18. Find the equations of the normal plane and the tangent line to the graph of the function $f(x, y) = 4x^2 + y^2 + 2$ at the point $(1,1)$.
19. Show that $(0,0)$ is a critical point of the function $f(x, y) = x^2 - y^2$ but that it does not give rise to a relative extremum of f.
20. Evaluate $\iint_R (2x - y)\, dA$ where R is the region bounded by the parabola $x = y^2$ and the straight line $x - y = 2$.
21. Show that the $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$ does not exist.
22. Find the area of the surface with equation $z = 2x + y^2$ that lies directly above the triangular region R in the xy-plane with vertices $(0,0), (1,1)$ and $(0,1)$.
23. Evaluate $\int_C (1 + xy)ds$, where C is the quarter-circle described by $r(t) = cost\boldsymbol{i} + sint\boldsymbol{j},\ 0 \le t \le \frac{\pi}{2}$. Also define a piecewise smooth curve.

SECTION C: Answer any *two* questions. Each carries *ten* marks.

24. a) Define a harmonic function. Show that $u = e^x cosy$ is harmonic in the xy –plane.

    b) Let f be a function of 2 variables. Prove that if f is differentiable at (a, b), then it is continuous at (a, b). Prove or disprove the converse.

25. State and prove Lagrange's Theorem. What is a Lagrange multiplier? Find the Lagrange multiplier if we need the maximum and minimum values of: $f(x, y) = x^2 - 2y$ subject to $x^2 + y^2 = 9$.
26. Evaluate $\iiint_T \sqrt{(x^2 + z^2)}dV$, where T is the region bounded by the cylinder: $x^2 + z^2 = 1$ and the planes $y + z = 2$ and $y = 0$.
27. Let $F(x, y, x) = 2xyz^2\boldsymbol{i} + x^2z^2\boldsymbol{j} + 2x^2yz\boldsymbol{k}$.
    a) Find a potential function f such that $F = \nabla f$.
    b) If F is a force field, find the work done by F in moving a particle along any path from (0,1,0) to (1,2,-1).

(2 x 10 = 20 Marks)