D3BEM2303 **(3 Pages)** **Reg.No:..................**

Name:....................

THIRD SEMESTER B. Sc. DEGREE EXAMINATION, NOVEMBER 2024
(Regular/ Improvement/ Supplementary)
ECONOMICS AND MATHEMATICS (DOUBLE MAIN)
GDEC3B03T: LINEAR PROGRAMMING AND PROBABILITY

Time: $2\frac{1}{2}$ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries two marks.
(Ceiling 25 Marks)

1. When a Linear Programming Problem is said to be unbounded?

2. Draw and shade the feasible region for the Linear Programming Problem :
Maximise $z = 2x + 4y$, subject to the constraints

$$\begin{aligned} x + 2y &\leq 5 \\ x + y &\leq 4 \\ x, y &\geq 0 \end{aligned}$$

3. Define slack and surplus variables.

4. Rewrite the Linear Programming Problem in standard form.
Minimise $2x_1 + x_2 + 4x_3$, subject to the constraints

$$\begin{aligned} -2x_1 + 4x_2 &\leq 4 \\ x_1 + 2x_2 + x_3 &\geq 5 \\ 2x_1 + 3x_3 &\leq 2 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

5. Define basic feasible solution to a Linear Programming Problem.

6. State fundamental theorem of Linear Programming.

7. State the necessary and sufficient condition for a basic feasible solution to an LPP to be optimal.

8. Write the dual of the Linear Programming Problem. Maximise $z = 5x_1 + 3x_2$, subject to

$$\begin{aligned} 3x_1 + 5x_2 &\leq 15 \\ 5x_1 + 2x_2 &\leq 10 \\ x_1, x_2 &\geq 0 \end{aligned}$$

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9. State the fundamental Theorem of duality.

P.T.O.

10. What is a non-degenerate basic feasible solution of a transportation problem.
11. Check whether the given transportation problem is balanced. If not, balance it.

	D	E	F	G	
A	10	12	16	13	20
B	15	17	15	9	30
C	20	23	12	9	30
	20	25	25	25	

12. Discuss pure and mixed strategies in a two person zero sum game.
13. State the axioms of probability.
14. A number is chosen form $\{1, 2, \cdots, 100\}$. What is the probability that it is divisible by either 3 or 5 or both?
15. A fair die is tossed eight times. What is the probability of exactly two 3's, three 1's and three 6's?

SECTION B: Answer the following questions. Each carries five marks. (Ceiling 35 Marks)

16. Show that set of feasible solutions to a Linear Programming Problem is a convex set.
17. Let $x_1 = 2, x_2 = 4, x_3 = 1$ be a basic feasible solution to the system of equations.

$$\begin{aligned} 2x_1 - x_2 + 2x_3 &= 2 \\ x_1 + 4x_2 &= 18 \end{aligned}$$

Reduce the given feasible solutions to a basic feasible solution.

18. State and prove weak duality theorem.
19. Obtain an initial basic feasible solution to the transportation problem using matrix minima method.

	D_1	D_2	D_3	D_4	Capacity
O_1	1	2	3	4	6
O_2	4	3	2	0	8
O_3	0	2	2	1	10
Demand	4	6	8	6	

20. Obtain the initial basic feasible solution to the following trasportation problem using North-West Corner Rule.

	D	E	F	G	
A	11	13	17	14	250
B	16	18	16	10	300
C	21	24	13	10	300
	200	225	275	250	

21. A Company wishes to assign 3 jobs to 3 machines in such a way that each job is assigned to some machine and no machine works on more than one job.

Cost matrix : $\begin{pmatrix} 8 & 7 & 6 \\ 5 & 7 & 8 \\ 6 & 8 & 7 \end{pmatrix}$, where a_{ij} = cost of assigning job i to machine j.

Formulate the LPP and find the minimum cost of making the assignment.

22. If we draw 8 cards form an ordinary deck of 52 cards, and 3 of them are spades, what is the probability that remaining 5 cards are also spades?

23. Show that 1) If $A \subseteq B$, then $P(A) \leq P(B)$.
2) $P(A \cup B) = P(A) + P(B) - P(AB)$.

SECTION C: Answer any two questions. Each carries ten marks.

24. Use Bin M- method to Maximise $z = 6x_1 + 4x_2$, subject to

$$\begin{aligned} 2x_1 + 3x_2 &\leq 30 \\ 3x_1 + 2x_2 &\leq 24 \\ x_1 + x_2 &\geq 3 \\ x_1, x_2 &\geq 0 \end{aligned}$$

25. Use dual simplex method to solve : Minimise $z = 3x_1 + x_2$, subject to

$$\begin{aligned} x_1 + x_2 &\geq 1 \\ 2x_1 + 3x_2 &\geq 2 \\ x_1, x_2 &\geq 0 \end{aligned}$$

26. Use Vogel's approximation method to obtain the initial basic feasible solution and hence find the optimal solution to the transportation problem.

	D_1	D_2	D_3	D_4	Supply
S_1	3	7	6	4	5
S_2	2	4	3	2	2
S_3	4	3	8	5	3
Demand	3	3	2	2	

27. A box contains 7 red and 13 blue balls. Two balls are selected at random and are discarded without their colors being seen. If a third ball is drawn randomly and observed to be red, what is the probability that both of the discarded balls were blue?

(2 x 10 = 20 Marks)