D3BMT1802(S4) **(PAGES 2)** **Reg.No:......................**

. **Name: ....**

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2023
(Supplementary 2018 Admission)
MATHEMATICS: Complementary Course to CHEMISTRY, PHYSICS & C.S.
AMAT3C03T: MATHEMATICS

Time 3 Hours **Maximum Marks: 80**

Part A: Answer all questions. Each carries one mark.

1. Solve the ODE $\frac{dy}{dx}+\frac{x}{y}=0$.
2. Define a separable differential equation. Give one example of such an equation.
3. Prove that $y\,dx+x\,dy$ is an exact differential equation.
4. Verify that the vectors $3\hat{i}-5\hat{j}+\hat{k}$ and $4\hat{i}+2\hat{j}-2\hat{k}$ are orthogonal.
5. Write the formula for the component of a force in a given direction. *Use suitable variables*.
6. Write the condition that the vectors **a**, **b** and **c** are linearly independent.
7. Define curl of a vector field.
8. Write the formula for work done by a force $\mathbf{f}(t)$ along a vector $\mathbf{r}(t)$, $a \le t \le b$.
9. State the Green's theorem in a plane.
10. Evaluate the double integral $\int_{-1}^{1}\int_{0}^{1} dx\,dy$.
11. State the Stoke's theorem.
12. Find the sum of coefficients of terms of $(1+x-x^2)^{15}$.

. **(12 x 1 = 12 Marks)**

Part B: Answer any nine questions. Each carries two marks.

13. Solve the initial value problem $\frac{dy}{dx}+y=0$, $y(0)=1$.
14. Verify that $y=e^{x^2/2}$ is a solution of the differential equation $\frac{dy}{dx}=x\,e^{x^2/2}$.
15. Solve the ODE $y'-y=e^{2x}$.
16. Write the differential equation of all curves passing through the point $(0,-1)$ and having slope at each point (x,y) as $\frac{y}{x}$.
17. For any two vectors **a** and **b**, prove that $|\mathbf{a}+\mathbf{b}|^2+|\mathbf{a}-\mathbf{b}|^2=2\left(|\mathbf{a}|^2+|\mathbf{b}|^2\right)$.
18. Write the equation of the level curve
for the function $f(x,y)=xy^2+x-3y$ passing through the point (2,-2).

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19. Find the gradient of the scalar function $f(x,y,z) = x^2y + 3x + yz^3$.

20. Evaluate the line integral of $f(x,y) = xy$ along the curve $y = \frac{1}{x}$ from (1,1) to $(2,\frac{1}{2})$.

21. Write the surface integral of $f(x,y) = xy$ over the hemisphere of $x^2 + y^2 + z^2 = 1$ above the x-y plane.

22. Evaluate $\int_0^1 \int_{-1}^1 \int_{-2}^2 xyz\,dx\,dy\,dz$.

23. Write the third term in the expansion of e^{2+3x}.

24. Write the coefficient of x^{12} in the expansion of $\log(1 - x^2/3)$.

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(9 x 2 = 18 Marks)

Part C: Answer any six questions. Each carries five marks.

25. Solve $2yy' + y^2 \sin x = \sin x$, $y(0) = \sqrt{2}$.

26. Let $\mathbf{f}(x,y,z) = (x^2 - y)\hat{\imath} + (y+z)\hat{\jmath} - yz^3\hat{k}$. Let $x = t^2$, $y = t^3$ and $z = t$. Find $\mathbf{f}'(t)$ and $\mathbf{f}''(t)$ at $t = 2$.

27. Prove that the gradient of a scalar field is a conservative vector field.

28. Test for path independence the line integral $\oint_{(0,0,0)}^{(1,1,1)} (2xyz^3 + 1)\,dx + (x^2z^3 + 4y)\,dy + 3x^2yz^2\,dz$.

29. Using the method of double integrals, determine the area of the surface $x^2 + y^2 + z = 1$, $z \geq 0$.

30. Simplify $1 + \frac{1}{4}x + \frac{1\cdot4}{4\cdot8}x^2 + \frac{1\cdot4\cdot7}{4\cdot8\cdot12}x^3 + \cdots$ to ∞.

31. What is the coefficient of x^n in the expansion of $e^{x/3}$?

32. Write the expansion of $(1 - 2x)\log(1 + x^2)$ as ascending powers of x. (Write the first four terms.)

33. Obtain the series for $\log(1 + 3x - 10x^2)$.

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(6 x 5 = 30 Marks)

Part D: Answer any two questions. Each carries ten marks.

34. Sketch the curves given by the equation $x^2 - y^2 = c$.
Find the slope of tangent at any point (x,y) on these curves.
Also find the equation of the orthogonal trajectories of the family $x^2 - y^2 = c$.

35. a) Find the length of the curve $\mathbf{r}(t) = \cos t\,\hat{\imath} + \sin t\,\hat{\jmath} + t\hat{k}, \quad 0 \leq t \leq 2\pi$.
    b) Find the equation of the tangent plane at (1,2,-2) on the surface $5x^2y + y^2z - x^3z = 4$.

36. Verify the divergence theorem for $\mathbf{F} = [x,y,z]$ and S is the surface of the sphere $x^2 + y^2 + z^2 = 9$.

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(2 x 10 = 20 Marks)