D3BEM2205 (PAGES 2) **Reg.No............................**

Name: ....

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2023

(Regular/Improvement/Supplementary)

ECONOMICS & MATHEMATICS (DOUBLE MAIN)

GDMT3B03T: MULTIVARIABLE CALCULUS

Time: 2 ½ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries *two* marks.
(Ceiling 25 Marks)

1. Show that $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$ does not exist.
2. Show that $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ if $f(x,y) = \sin(xy) + x^3y^5 + x^4 - y^2 + 8$.
3. Find $\frac{dy}{dx}$ if $x^3 + xy + y^2 = 4$.
4. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $2x^3z - 3xy^2 + yz + 8 = 0$.
5. Find the critical points of $f(x,y) = x^4 + 3x^2y^3 + 4x^3y^2 + 5xy + 11$.
6. Define relative maximum value and relative minimum value.
7. State Lagrange's Theorem.
8. State The Extreme Value Theorem for functions of two variables.
9. Evaluate $\int_1^2 \int_0^1 3x^2y \, dxdy$.
10. State Fubini's Theorem for Rectangular Regions.
11. Define the Jacobian of the transformation.
12. Find the gradient vector field of $f(x,y,z) = x^2 + y^2 + 2x^3z^4$.
13. Define Divergence of a Vector Field. When will we call a field solenoidal?
14. Find the curl of the vector field $\boldsymbol{F}(x,y,z) = x^2yz\boldsymbol{i} + y^2\boldsymbol{j} + xyz\boldsymbol{k}$.
15. Determine whether the vector field $\boldsymbol{F}(x,y) = 2xy^2\boldsymbol{i} + x^2y\boldsymbol{j}$ is conservative.

SECTION B: Answer the following questions. Each carries *five* marks.

(Ceiling 35 Marks)

16. Find the second-order partial derivatives of $f(x,y) = x^3y^3 + x^6 + y^4 + 8 + e^{xy}$.
17. Let $w = x^2y - xy^3$, where $x = cost$ and $y = e^t$. Find $\frac{dw}{dt}$ and also find the value at $t = 0$.
18. Find the absolute extreme values of $f(x,y) = 2x^2 + y^2 - 2y + 1$, subject to the constraint $x^2 + y^2 \leq 4$.
19. Show that the point is a critical point of $f(x,y) = y^2 - x^2$, but that it does not give rise to a relative extremum of f.

(PTO)

20. Find the level curve of the function $f(x, y) = x^2 + y^2$ passing through the point P(2,0). Also find the gradient of f at P. Sketch the level curve and the gradient vector at P.

21. Evaluate $\int_0^1 \int_y^1 \frac{sinx}{x} dxdy$.

22. Use a double integral to find the area enclosed by one loop of the three leaved rose $r = \sin 3\theta$.

23. Find the work done by the force field $\boldsymbol{F}(x, y, z) = -y\boldsymbol{i} + x\boldsymbol{j} + z\boldsymbol{k}$ in moving a particle along the helix C described by the parametric equations $x = \cos t, y = \sin t, z = t$ from (1,0,0) to $(0,1,\frac{\pi}{2})$.

SECTION C: Answer any *two* questions. Each carries *ten* marks.

24. Find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ where $w = x^2 + y^2 + z^2$, where $x = e^v \cos u, y = e^v \sin u, z = e^v u$.

25. Find the relative extrema of $f(x, y) = x^3 + y^2 - 2xy + 7x - 8y + 2$.

26. Find the volume of the solid lying under the elliptic paraboloid $z = 8 - 2x^2 - y^2$ and above the rectangular region $R = \{(x, y)/0 \leq x \leq 1, 0 \leq y \leq 2\}$.

27. Let $\boldsymbol{F}(x, y) = 2xy\boldsymbol{i} + (1 + x^2 - y^2)\boldsymbol{j}$.
    a. Show that F is conservative and find a potential function f such that $F = \nabla f$.
    b. If F is a force field, find the work done by F in moving a particle along any path from $(1,0)$ to $(2,3)$.

(2 x 10 = 20 Marks)