

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2023
(Regular/Improvement/Supplementary)

MATHEMATICS: COMPLEMENTARY COURSE FOR PHYSICS, CHEMISTRY & C S

GMAT3C03T: MATHEMATICS - 3

Time: 2 Hours

Maximum Marks: 60

SECTION A: Answer the following questions. Each carries *two* marks.
(Ceiling 20 Marks)

1. Define a linear differential equation and write its general form.
2. Verify that the function $y = ce^{-x} + x^2 - 2x$ is a solution of the differential equation $y' + y = x^2 - 2$.
3. Define differential of a function $u(x, y)$. What is the differential of $x + x^2y^3 = c$.
4. Is $2xy \, dy = (x^2 + y^2)dx$ exact? If not, find the integrating factor.
5. Verify by substitution that the function $y = 1 + \sin x$ is a solution of $y'' + y = 1$.
6. Let $\vec{a} = [2, 1, 0]$ and $\vec{b} = [3, 2, 1]$; Find $3\vec{a} \times 5\vec{b}$.
7. Find velocity and acceleration of $\vec{r}(t) = [t, t^2, t^3]$ at P: (1, 1, 1).
8. Prove that $3y^4z^2 \, i + 4x^3z^3 \, j - 3x^2y^2 \, k$ is solenoidal.
9. Define curl of a vector field \vec{v} .
10. State Stoke's theorem.
11. Give parametric and vector representation of sphere with radius a ; $a > 0$ and center at origin.
12. Find unit normal vector of the cone $g(x, y, z) = -z + \sqrt{x^2 + y^2} = 0$.

SECTION B: Answer the following questions. Each carries *five* marks.
(Ceiling 30 Marks)

13. Find the orthogonal trajectories of the family of parabolas $y = cx^2$.
14. By the method of undetermined coefficients, find a particular solution of $y'' - 5y' + 6y = 2e^t$.
15. Find the components of the vector \vec{V} with initial point P and terminal point Q.

Find $|\vec{V}|$, and the unit vector \vec{U} in the direction of \vec{V} . P: (-3.0, 4.0, -0.5), Q: (5.5, 0, 1.2)

(PTO)

16. Find Grad f . Where $f = (x + 1)(2y - 1)$.

17. Find the value of the line integral when $\vec{F}(\mathbf{r}) = [z, x, y]$ and C is the helix

$$\vec{r}(t) = [\cos t, \sin t, 3t]; 0 \leq t \leq 2\pi.$$

18. Find the volume of the region between the cylinder $z = y^2$ and the XY plane that is bounded by the plane $x = 0, x = 1, y = -1, y = 1$.

19. Find the moment of inertia of a rectangle solid $-\frac{a}{2} \leq x \leq \frac{a}{2}, -\frac{b}{2} \leq y \leq \frac{b}{2}, -\frac{c}{2} \leq z \leq \frac{c}{2}$ of constant density $\sigma = 1$ about X axis.

SECTION C: Answer any *one* question. Each carries *ten* marks.

20. a.) Verify that the functions $y_1 = x^{-\frac{1}{2}}, y_2 = x^{\frac{3}{2}}$ are linearly independent and form a basis of solutions of the ODE $4x^2y'' - 3y = 0$.

b.) Solve the IVP $4x^2y'' - 3y = 0$ $y(1) = 3$ $y'(1) = 2.5$.

21. Compute the flux of water through $S: y = x^2, 0 \leq x \leq 2, 0 \leq z \leq 3$, if the velocity vector is $\vec{V} = \vec{F} = [3z^2, 6, 6xz]$.

(1 x 10 = 10 Marks)