**D3BST2201** (PAGES 2) **Reg.No..............................**

**Name: ..............................**

**THIRD SEMESTER DEGREE EXAMINATION, NOVEMBER 2023**

**(Regular/Improvement/Supplementary)**

**STATISTICS COMPLEMENTARY COURSE FOR MATHEMATICS & C S**

**GSTA3C03T: PROBABILITY DISTRIBUTIONS AND SAMPLING THEORY**

**Time: 2 Hours** **Maximum Marks: 60**

**SECTION A: Answer the following questions. Each carries *two* marks.**

**(Ceiling 20 Marks)**

1. Derive probability mass function of a Binomial distribution. For a Binomial distribution with mean = 6 and variance= 2, find$P(X > 1)$.
2. Obtain the moment generating function of a Geometric distribution.
3. If X follows Beta distribution of type 1 with parameters *m* and *n,* what is its mean?
4. What are the important properties of Normal distribution?
5. If $Z \rightarrow N(0,1)$ find $P(-2 < Z < 2.5)$.
6. State Bernoulli 's law o f large numbers.
7. If $X \rightarrow N(16, 2)$, find an upper bound for $P$ (|X - 16 | > 6) using Chebychev's inequality.
8. What are the important methods of random sampling?
9. What is cluster sampling?
10. Distinguish between parameter and statistic.
11. What are the characteristics of F distribution?
12. Identify the distribution of the ratio of the squares of two independent standard normal random variables.

**SECTION B: Answer the following questions. Each carries *five* marks.**

**(Ceiling 30 Marks)**

13. If X and Y are independent Poisson variates such that P[X = 1] = P[X = 2] and P[Y = 2] = P[Y = 3] find the variance of X – 2Y.
14. If X follows Beta distribution of second kind with parameters m and n. show that $\frac{1}{1+X}$ follows beta distribution of first kind with parameters m and n.
15. If $X \rightarrow N(\mu, \sigma)$, show that Y = $x^2$ follow gamma distribution with parameters ( ½ ,½)·

**(PTO)**

16. If $X \to N(20, 25)$, find the P (|X-20| >5).

17. Let $X_i$ assumes the values $i^\alpha$ and $-i^\alpha$ with equal probability. Check whether the law of large numbers hold to the independent variables $X_1$, $X_2$, … for $\alpha < \frac{1}{2}$.

18. Explain systematic sampling.

19. If X is distributed as $f(x) = \frac{1}{\theta}$ , $0 < x < \theta$ , show that $-2\log(\frac{x}{\theta})$ follows chi - square distribution with 2 degrees of freedom.

**SECTION C: Answer any *one* question. Each carries *ten* marks.**

20. (i) State Chebyshev's inequality.

(ii) Let X be a random variable taking values – 1, 0 , +1 with probabilities $\frac{1}{8}, \frac{6}{8}, \frac{1}{8}$ respectively. Find, using Chebyshev's inequality, the upper bound of the probability $P[|X| \geq 1]$.

21. (i) Define t statistic and give its pdf.

(ii) If $\bar{x}$ and $S^2$ are the mean and variance of samples taken from $N(\mu, \sigma)$, $\sigma$ is unknown. Find the distribution of $T = \frac{\bar{x} - \mu}{S/\sqrt{n-1}}$.

**(1 x 10 = 10 Marks)**