D3BMT2201 (PAGES 2) **Reg.No.............................**

Name: ....

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2023

(Regular/Improvement/Supplementary)

MATHEMATICS

GMAT3B03T: GEOMETRY AND VECTOR CALCULUS

Time: 2 ½ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries *two* marks.

(Ceiling 25 Marks)

1. Find the focus and directrix of the parabola $y^2 + 6x = 0$. And make a sketch of the parabola.
2. Find the slope of the tangent line to the curve $x = 2\sin\theta\,, y = 3cos\theta$ at the point corresponding to $\theta = \frac{\pi}{4}$.
3. Sketch the curve represented by $x = \sqrt{t}$ and $y = t$.
4. Determine whether the planes $x - y + 2z = 5$ and $-3x + 3y - 6z = 11$ are parallel or not?
5. Find the angle between the planes defined by $3x - y + 2z = 1$ and $2x + 3y - z = 4$.
6. The point $(1, \sqrt{3}, 5)$ is expressed in rectangular coordinates. Find its cylindrical coordinates.
7. Sketch the graph of the cylinder with the equation $x^2 + y^2 = 4$.
8. Define vector valued function.
9. Find the interval(s) on which the vector function defined by $\vec{r}(t) = \frac{\cos t - t}{t}\vec{\imath} + \frac{\sqrt{t}}{1+2t}\vec{\jmath} + te^{-1/t}\vec{k}$is continuous.
10. Define radius of curvature of a plane curve?
11. Evaluate the integral $\int (t\vec{\imath} + 2t^2\vec{\jmath} + 3\vec{k})\,dt$.
12. Define graph of a function of two variables.
13. Evaluate $\lim_{(x,y,z)\to(\frac{\pi}{2},0,1)} \frac{e^{2y}(\sin x + \cos y)}{1+z^2+y^2}$.
14. State Clairaut's theorem.
15. Let $f(x,y) = x^2 + y^2$. Find $f_x(2,1)$ and $f_y(2,1)$.

SECTION B: Answer the following questions. Each carries *five* marks.
(Ceiling 35 Marks)

16. Find the area of the surface obtained by revolving the curve $r = 4cos\theta$ about the polar axis.
17. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ where $x = \sqrt{t}$ and $y = {}^{1}/_{t}$.
18. Find an equation of the plane that is orthogonal to the plane $3x + 2y - 4z = 7$ and contains the line of intersection of the planes $2x - 3y + z = 3$ and $x + 2y - 3z = 5$.

(PTO)

19. Write the equation $x^2 + y^2 = 4y$ in cylindrical and in spherical coordinates.

20. Sketch the curve defined by the vector function $\vec{r}(t) = (2+t)\vec{\imath} + (3-2t)\vec{\jmath} + (2+4t)\vec{k}$, $0 \le t \le 1$.

21. Find the curvature of the curve $y = \sin 2x$.

22. Suppose z is a differentiable function of x and y that is defined implicitly by $x^2 + y^3 - z + 2yz^2 = 5$. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

23. Find the second partial derivatives of the function $f(x,y) = x^4 - 2x^2y^3 + y^4 - 3x$.

SECTION C: Answer any *two* questions. Each carries *ten* marks.

24. Consider the cardioid $r = 1 + cos\theta$.

a) Find the slope of the tangent line to the cardioid at the point where $\theta = \frac{\pi}{6}$

b) Find the points on the cardioid where the tangent lines are horizontal and where the tangent lines are vertical.

25. Find the point of intersection of the lines.

$$L_1 \ : \frac{X-2}{4} = \frac{Z-1}{-1}, \qquad y = 3$$

$$L_2 \ : \frac{x-2}{2} = \frac{y-3}{2} = z-1$$

Also find the angle between the two lines.

26. A moving object has an initial position and an initial velocity given by the vectors $\vec{r}(0) = \vec{\imath} + 2\vec{\jmath} + \vec{k}$ and $\vec{v}(0) = \vec{\imath} + 2\vec{k}$. Its acceleration at time t is $\vec{a}(t) = 6t\vec{\imath} + \vec{\jmath} + 2\vec{k}$. Find its velocity and position at time t.

27. (i) Show that $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$ does not exist.

(ii) Show that $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ does not exist.

(2 x 10 = 20 Marks)