Name: ....

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2023

COMPUTER SCIENCE AND MATHEMATICS (DOUBLE MAIN)

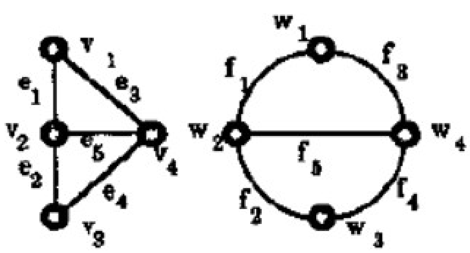
GDMA3A01T: BASIC LOGIC, BOOLEAN ALGEBRA AND GRAPH THEORY

Time: 2 ½ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries *two* marks.

(Ceiling 25 Marks)

1. Define negation of a proposition with truth table.
2. Verify $p \wedge q \equiv q \wedge p$.
3. Define Euler graph. Give one example.
4. Negate the proposition, where x is an arbitrary integer $(\exists x)(x^2 \neq 5x - 6)$
5. Explain existence proof. What are two kinds of it?
6. Define a bridge. Give one example.
7. When do we say that two ordered sets are isomorphic?
8. State the well-ordering theorem.
9. Give an example of a bounded lattice.
10. When do we say that a lattice is complemented?
11. Are the following graphs isomorphic? Justify your statement.



12. Define the union of two subgraphs of a graph and give one example.
13. Let G_1 and G_2 be two plane graphs which are both redrawings of the same planar graph G. Prove that G_1 and G_2 have the same number of faces.
14. Define a Jordan curve. Give one example.
15. State Whitney's theorem.

SECTION B: Answer the following questions. Each carries *five* marks.

(Ceiling 35 Marks)

16. Rewrite each sentence symbolically, where UD consists of real numbers.
    - (i) The product of any two real numbers x and y is positive .
    - (ii) There are real numbers x and y such that x=2y
    - (iii) For each real number x, there is some real number y such that x · y =x
    - (iv) There is a real number x such that x +y =y for every real number y
17. Prove that there is a prime number > 3.
18. Prove that every linearly ordered set is a lattice.

19. Let L be a finite complemented distributive lattice. Then show that every element a in L is the join of a unique set of atoms.

20. Draw a graph having the following matrix as its adjacency matrix.

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 2 \\ 0 & 2 & 1 & 2 \\ 0 & 2 & 2 & 1 \end{bmatrix}$$

21. Let T be a tree with at least two vertices and let $P = u_0u_1 \ldots u_n$ be a longest path in T.Then prove that both u_0 and u_n have degree 1.

22. Let G be a graph with n vertices,where $n \geq 2$.Prove that G has at least two vertices which are not cut vertices.

23. Prove that a simple graph G is Hamiltonian iff its closure $c(G)$ is Hamiltonian.

SECTION C: Answer any *two* questions. Each carries *ten* marks.

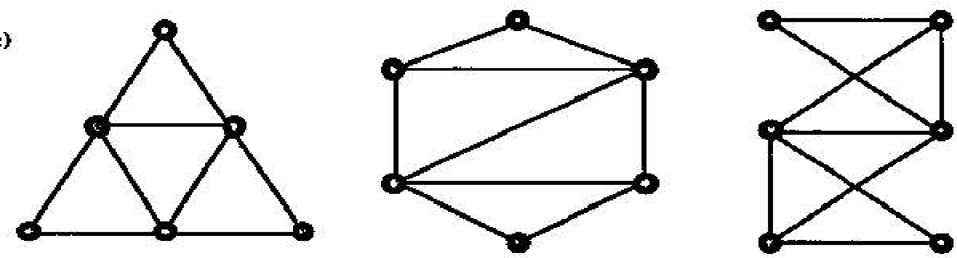
24. Determine whether or not each a tautology $(i)[(p \rightarrow q) \wedge (\sim q)] \rightarrow \sim p$

$(ii)[p \wedge (p \rightarrow q)] \rightarrow q$

25. a) Define isomorphism.

b) Which of the following graphs are isomorphic pairs. Justify your statement.

(c)



26. a) Let G be a graph with n vertices, where n≥2. Then prove that G has at least two vertices which are not cut vertices.

b) State Whitney's theorem and prove that if u and v are two vertices of the 2 - connected graph G, then there is a cycle C of G passing through both u and v.

27. Define consistent enumeration of a finiteposet and prove that there exists a consistent enumeration for any finite poset A.

(2 x 10 = 20 Marks)