

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2023

HONOURS IN MATHEMATICS

GMAH3B13T: STATISTICAL INFERENCE

Time: 3 Hours

Maximum Marks: 80

PART A: Answer *all* the questions. Each carries *one* mark.

Choose the correct answer.

- The standard error of a sample mean of a random sample of size n from $N(\mu, \sigma)$ is:
(a) One (b) Zero (c) $\frac{\sigma^2}{n}$ (d) $\frac{\sigma}{\sqrt{n}}$
- The maximum likelihood estimates are necessarily
(a) Biased. (b) Sufficient. (c) Consistent. (d) Most efficient.
- For a fixed confidence coefficient $(1-\alpha)$, the most preferred confidence interval for the parameter θ is:
(a) With largest width (b) With shortest width
(c) Any width (d) All the above
- Accepting alternative hypothesis when it is false is:
(a) Type I error (b) Standard error (c) Sampling error (d) Type II error
- Analysis of variance utilizes..... test.
(a) F - test (b) t - test (c) χ^2 test (d) Z-test

Fill in the Blanks.

- A function for estimating a parameter is called.....
- The difference between the expected value of an estimator and the value of the corresponding parameter is known as.....
- Probability of type I error is called.....
- Analysis of variance utilizes... .. test.
- The technique of analysis of variance is developed by..... ..

(10 x 1 = 10 Marks)

PART B: Answer any *eight* questions. Each carries *two* marks.

- Define sampling distribution. Give an example.
- Define t distribution.
- If $X \rightarrow F(m, n)$. Show that $\frac{1}{X} \rightarrow F(n, m)$.
- What are the properties of a good estimator?
- State Neyman factorization theorem.
- Highlight the applications of F - test.
- What are the sufficient conditions for consistency?
- Define statistical hypothesis. Give an example.
- What are the errors in testing of hypothesis? Explain.
- Mention the applications of t distribution.

(8 x 2 = 16 Marks)

(PTO)

PART C: Answer any six questions. Each carries four marks.

21. Obtain the sampling distribution of the sample mean of a random sample of size n from a normal population with parameters μ and σ^2 .
22. Define chi - square distribution. State and prove the additive property of chi - square distribution.
23. Prove that square of a Student's t random variable is F random variable.
24. Show that sample mean of a random sample of size n from $f(x; \theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}}$; $x \geq 0$ is consistent for θ .
25. Let $\bar{x} = 25$ be the mean of a random sample of size 25 and $S = 4$ be the sample standard deviation. Obtain the 90% confidence interval for the mean of a normal population.
26. Define the following
(i) Types of errors (ii) Significance level (iii) Critical region (iv) Parametric test
27. A random sample of 100 villages was taken from Malappuram district and the average population per village was found to be 300 with standard deviation of 40. Another random sample of 100 villages from the same district gave an average population of 450 per village with standard deviation of 50. Is the difference between the averages of the two samples statistically significant?
28. A certain stimulus administered to each of the 12 patients resulted in the following increase in blood pressure.
5, 2, 8, -1, 3, 0, -2, 1, 5, 0, 4 and 6
Can it be concluded that the stimulus will, in general be accompanied by an increase in blood pressure?

(6 x 4 = 24 Marks)

PART D: Answer any two questions. Each carries fifteen marks.

29. Obtain the MLE's of μ and σ^2 , where the samples are taken from $N(\mu, \sigma)$.
30. (i) In a sample of 20 persons from a town it was found that 4 are suffering from TB. Find a 95% confidence interval for the proportion of TB persons in the town.
(ii) A random sample of 20 bullets produced by a machine shows an average diameter of 3.5mm and a standard deviation of 0.2mm. Assume that the diameter measurement follows normal with usual parameters. Obtain 95% confidence interval for variance.
31. Two random samples are drawn from two normal populations and the following results were obtained.

Sample I :	16	17	18	19	20	21	22	24	26	27
Sample II :	19	22	23	25	26	28	29	30	31	32

Obtain estimates of the variances of populations and test whether the populations have the same variance.

(2 x 15 = 30 Marks)