

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2023

HONOURS IN MATHEMATICS

GMAH3B12T: DIFFERENTIAL EQUATIONS

Time: 3 Hours

Maximum Marks: 80

PART A: Answer *all* the questions. Each carries *one* mark.

Multiple choice questions:

- The order of the differential equation $(t^2 + 1)y'''' + 2t y' + 2y = t + 6$ is.....
 - 1
 - 2
 - 3
 - None of the above
- The Wronskian of $\cos t$ and $\sin t$ is.....
 - 0
 - 1
 - 2
 - 1
- A matrix is said to be Hermitian if
 - $A^2 = A$
 - $A^* = A$
 - $A^T = A$
 - None of the above
- Let $f(t) = \begin{cases} t & , 0 \leq t \leq 1 \\ 3 - t & , 1 < t \leq 2 \\ 1 & , 2 < t \leq 3 \end{cases}$ then f is.....
 - Continuous on $[0, 3]$
 - Piecewise continuous on $[0, 3]$
 - Neither continuous nor piecewise continuous on $[0, 3]$
 - None of the above
- The fundamental period of $\sin \pi x/L$ is.....
 - l
 - L
 - $2L$
 - None of the above

Fill in the blanks:

- An example of an exact differential equation is
- $W(t^2, 1 + t^2) = \dots\dots\dots$
- The characteristic equation corresponding to the differential equation $3y'' + 6y' + 2y = 0$ is
- The Dirac delta function is
- The Euler-Fourier formulas to find a_0 of a periodic function $f(x)$ with period $2L$ is

(10 x 1 = 10 Marks)

PART B: Answer any *eight* questions. Each carries *two* marks.

- Determine whether the functions t and t^{-1} are linearly dependent or linearly independent on an arbitrary interval.
- Find Eigen values of the matrix $\begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix}$
- Graph the unit step function $u_c(t)$.

(PTO)

14. Find $\mathcal{L}^{-1}\left(\frac{3!}{(s-2)^4}\right)$.

15. Solve the boundary value problem $y'' + 2y = 0$, $y(0) = 0$, $y(\pi) = 0$.

16. Find $\mathcal{L}(te^{2t})$.

17. Find the value of r if $y = e^{rt}$ is a solution of $y'' + y' - 6y = 0$.

18. Find the Fourier coefficient a_0 for the function

$$f(x) = x, -1 \leq x \leq 1;$$

$$f(x+2) = f(x)$$

19. Find the Wronskian of e^{2t} and te^{2t} .

20. Find $t^2 * 2t$.

(8 x 2 = 16 Marks)

PART C: Answer any six questions. Each carries four marks.

21. Find the general solution of $2y'' - 3y' + y = 0$.

22. Find $\mathcal{L}(\sin at)$ using definition.

23. Sketch the graph of $y = h(t)$, where $h(t) = u_\pi(t) - u_{2\pi}(t)$, $t \geq 0$.

24. Solve $y' + y = 5 \sin 2t$.

25. Find $t^2 * \sin t$.

26. Verify that $y_1(t) = t^{-2}$ and $y_2(t) = t^{-2} \ln t$ are solutions of

$$t^2 y'' + 5t y' + 4y = 0, t > 0.$$

27. Solve the boundary value problem $y'' + 2y = x$, $y(0) = 0$, $y(\pi) = 0$.

28. Find the Wronskian of two solutions of $\cos t y'' - \sin t y' - ty = 0$.

(6 x 4 = 24 Marks)

PART D: Answer any two questions. Each carries fifteen marks.

29. a) Solve $(3xy + y^2) + (x^2 + xy)y' = 0$.

b) Use Picard's method, solve $y' = 2(y + 1)$, $y(0) = 0$.

30. a) Solve the i.v.p. $6y'' - 5y' + y = 0$, $y(0) = 4$, $y'(0) = 0$

b) Verify that the functions $y_1(t) = e^t$ and $y_2(t) = te^t$ are solutions of $y'' - 2y' + y = 0$. Do they constitute a fundamental set of solutions?

31. Find the Fourier series for the function $f(x) = \begin{cases} -1, & -2 \leq x < 0 \\ 1, & 0 \leq x < 2 \end{cases}$

$$f(x+4) = f(x)$$

(2 x 15 = 30 Marks)