D3BHM2202 (PAGES 2) **Reg. No............................**

Name: ....

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2023

HONOURS IN MATHEMATICS

GMAH3B11T: CALCULUS III

Time: 3 Hours **Maximum Marks: 80**

PART A: Answer *all* the questions. Each carries *one* mark.

Choose the correct answer.

1. The spherical Coordinates of the point $(1,0,-\sqrt{3})$ given in rectangular coordinates is__________.
   A) $\left(2,\frac{\pi}{6},0\right)$ B) $(2,\frac{5\pi}{6},0)$ C) $(2,\frac{-\pi}{6},0)$ D) $(2,\frac{-5\pi}{6},0)$

2. The symmetric equations of the line L passing through the point $P_0(-1,2,3)$ and parallel to the vector $\boldsymbol{v} =< 2,1,-1 >$ are______________.
   A) $\frac{x+1}{2} = y-2 = 3-z$ B) $\frac{x-1}{2} = \frac{y-2}{2} = 3-z$
   C) $\frac{x+1}{2} = y-2 = z-3$ D) $\frac{x+1}{2} = y-2 = 3-z$

3. The standard form of the equation of a plane containing the point $P_0(x_0,y_0,z_0)$ and having the normal vector $\boldsymbol{n} = < a,b,c >$ is_______________.
   A) $a/(x-x_0) + b/(y-y_0) + c/(z-z_0) = 0$
   B) $a(x+x_0) + b(y+y_0) + c(z+z_0) = 0$
   C) $(a-x_0) + (b-y_0) + (c-z_0) = 0$
   D) $a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

4. Let $f(x,y) = x^2y - 2x + 2y$. The value of $f(1,2)$ is_______________.
   A) 2 B) 6 C) 4 D) -2

5. $\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^2+y^2} =$ _________.
   A) 1 B) 0 C) 2 D) Limit does not exist.

Fill in the Blanks.

6. A _____________function is the quotient of two polynomial functions
7. If $f(x,y) = x\cos xy^2$ then $f_x =$ ______________.
8. If u is a function of two variables x and y, the partial differential equation $u_{xx} + u_{yy} = 0$ is called _______________.
9. The domain of the vector function $\mathbf{r}(t) = 2t\mathbf{i} + \frac{2}{t}\mathbf{j}$ is_______________.
10. Suppose that C is a plane curve with curvature K at the point P. Then the reciprocal of the curvature, $\rho = \frac{1}{K}$, is called the _____________ of C at P .

(10 x 1 = 10 Marks)

PART B: Answer any *eight* questions. Each carries *two* marks.

11. Find the velocity vector and acceleration vector of an object that moves along the plane curve described by the position vector $\mathbf{r}(\mathrm{t}) = 2\ \cos \mathrm{t}\,\mathbf{i} + 3\sin \mathrm{t}\,\mathbf{j}$.
12. Find $\int \mathbf{r}(t)dt$ if $\mathbf{r}(\mathrm{t}) = (2\mathrm{t}+1)\mathbf{i} + \sin 2\mathrm{t}\mathbf{j} + \mathrm{e}^{5\mathrm{t}}\mathbf{k}$.
13. Show that $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{2x^2+y^2}$ does not exist.

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14. Find f_x and f_y if $f(x,y) = y \sin x^2 y$.

15. State Clairaut's Theorem.

16. Evaluate $\int_0^2 \int_1^3 5xy^3 dxdy$.

17. Let L_1 be the line with parametric equations $x = 2 + 2t$, $y = 3 - 3t$, $z = 4 + t$ and L_2 be the line with parametric equations $x = 4 - 4t, y = 4 + 4t, z = -4 + 4t$. Show that the lines L_1 and L_2 are not parallel to each other.

18. Find the distance between the point $(2, -1, -3)$ and the plane $2x + 3y - z = 2$.

19. Sketch the graph of the cylinder $x^2 + y^2 = 4$.

20. Find the directional derivative of $f(x,y) = e^{2x} \sin y$ at the point $(0, \frac{\pi}{4})$ in the direction of $\mathbf{v} = 3\mathbf{i} + 2\mathbf{j}$. .

(8 x 2 = 16 Marks)

PART C: Answer any *six* questions. Each carries *four* marks.

21. Prove that if a function f is differentiable at the point (x,y) and if $\nabla f(x,y) = 0$, then $\mathrm{D}_{\mathbf{u}}\mathrm{f}(\mathrm{x},\mathrm{y}) = 0$ for every $\mathbf{u}$.

22. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $4x^2 z - 2x^2 y + xz - 5 = 0$.

23. Let $f(x,y,z) = ze^{xy}$. Compute f_{xyz} and f_{zyx}.

24. Sketch the curve defined by the vector function $\mathbf{r}(\mathrm{t}) =< 2\cos \mathrm{t}, -3\sin \mathrm{t} >, \quad 0 \leq \mathrm{t} \leq 2\pi$.

25. Find the interval(s) on which the vector function **r** defined by **r**(t) = t**i**+$\left(\frac{1}{t-1}\right)$**j**+$\sqrt{t}$ **k** is continuous.

26. Find the anti derivative of $r'(t) = \sin t\, i + \sqrt{t} j + e^{3t} k$ satisfying the initial condition $r(0) = i + 2j + 4k$.

27. Find the angle between the two planes defined by $2x - 3y + z = 1$ and $x + y - z = 4$.

28. Find the second order derivatives of the function $h(x,y) = \tan^{-1}\frac{y}{x}$ and check whether $h(x,y)$ is a harmonic function.

(6 x 4 = 24 Marks)

PART D: Answer any *two* questions. Each carries *fifteen* marks.

29. Identify and sketch the surface $12x^2 - 3y^2 + 4z^2 + 12 = 0$.

30. a) Suppose that **r** is integrable on $[a,b]$ and that c is a constant vector.

    Prove that $\int_a^b c.r(t)dt = c.\int_a^b r(t)dt$.

    b) Verify this property directly for the vector function $\mathbf{r}(\mathrm{t}) = \cos \mathrm{t}\,\mathbf{i} + \sin \mathrm{t}\,\mathbf{j} + \mathrm{t}\mathbf{k}$, $\mathrm{c} = 4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $a = 0, b = \pi$..

31. Let $f(x) = \begin{cases} 1 & if\ x \geq 0 \\ -1 & if\ x < 0 \end{cases}$. Prove that $\lim_{x \to 0} f(x)$ does not exist.

(2 x 15 = 30 Marks)