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D3BMT1802(S3) (PAGES 2) Reg.No:......................

Name: ....

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2022
(Supplementary 2018 Admission)
MATHEMATICS: Complementary Course to CHEMISTRY, PHYSICS & C.S.
AMAT3C03T: MATHEMATICS

Time 3 Hours **Maximum Marks: 80**

Part A: Answer all questions. Each carries 1 mark.

1. Solve the ODE $\frac{dy}{dx} - \frac{y}{x} = 0$.
2. Write the integrating factor of the linear differential equation $x\frac{dy}{dx} + y = \sin x$.
3. Define an exact differential equation. Give one example of such an equation.
4. Find λ such that the vectors $2\hat{i} + \hat{j} - \lambda\hat{k}$ and $\hat{i} + 3\hat{j} + \hat{k}$ are orthogonal.
5. Write the formula for work done by a force making a displacement. *Use suitable variables.*
6. Write the condition that the vectors **a**, **b** and **c** are coplanar.
7. Define divergence of a vector field.
8. Write the formula for circulation of a flow given as $\mathbf{f}(t)$ along a curve $\mathbf{r}(t)$, $a \leq t \leq b$.
9. State the Green's theorem in a plane.
10. Evaluate the double integral $\int_0^1 \int_0^2 dx\,dy$.
11. State the Divergence theorem.
12. Find the sum of coefficients of terms of $(3 - 5x + 2x^2)^{11}$.

(12 × 1 = 12 **Marks.)**

Part B: Answer any 9 questions. Each carries 2 marks.

13. Solve the initial value prolem $\frac{dy}{dx} - xy = 0$, $y(0) = 2$.
14. Verify that $y = \tan(x + c)$ is a solution of the differential equation $\frac{dy}{dx} = 1 + y^2$.
15. Solve the ODE $y' + y\tan x = \sin 2x$.
16. Write the differential equation of all curves passing through the point $(1,1)$ and having slope at each point (x,y) as $2xy$.
17. If **a**, **b** and **a** + **b** are unit vectors, then find the angle between the vectors **a** and **b**.
18. Write the equation of the level curve for the function $f(x,y) = x^2y - 2x - y$ passing through the point (3,-1).
19. Find the gradient of the scalar function $f(x,y,z) = x^2y + 3x + yz^3$.

(P.T.O.)

20. Evaluate the line integral of $f(x,y) = 2x + 3y$ along the curve $y = x^2$ from (0,0) to (1,1).

21. Write the surface integral of $f(x,y) = x + y$ over the hemisphere of $x^2 + y^2 + z^2 = 1$ above the x-y plane.

22. Evaluate $\int_0^1 \int_0^2 \int_0^4 xyz\,dx\,dy\,dz$.

23. Write the third term in the expansion of $(4-x)^{-1/2}$.

24. Write the coefficient of x^7 in the expansion of $\log(1-3x/4)$.

(9 × 2 = 18 **Marks.**)

Part C: Answer any 6 questions. Each carries 5 marks.

25. Solve $y' + (x+1)y = e^{x^2}y^3$, $y(0) = 0.5$.

26. Let $\mathbf{f}(x,y,z) = (x+2y)\hat{\imath} + yz\hat{\jmath} + 3xz^2\hat{k}$. Let $x = 1+t$, $y = t^2$ and $z = t$. Find $\mathbf{f}'(t)$ and $\mathbf{f}''(t)$ at $t = 2$.

27. Define a scalar field and a vector field. Give one examle for each.
Find the divergence of $\mathbf{f}(x,y,z) = x^3yz^2\hat{\imath} + xy^2z\hat{\jmath} + (x-y)z^3\hat{k}$.

28. Test for path independence the line integral $\oint_{(0,0,0)}^{(1,1,1)} (3x^2y^2z + z^2)\,dx + (2x^3yz + 1)\,dy + (x^3y^2 + 2xz)\,dz$.

29. Using the method of double integrals, determine the area of the hemisphere $x^2 + y^2 + z^2 = 1$, $z \geq 0$.

30. Simplify $1 + \frac{1}{3}x + \frac{1\cdot 3}{3\cdot 6}x^2 + \frac{1\cdot 3\cdot 5}{3\cdot 6\cdot 9}x^3 + \cdots$ to ∞.

31. What is the coefficient of x^n in the expansion of $(1-x/2)^{-1}$?

32. Write the expansion of $(1+3x)e^{2+x}$ as ascending powers of x. (Write the first four terms.)

33. Obtain the series for $\log(1-5x+6x^2)$.

(6 × 5 = 30 **Marks.**)

Part D: Answer any 2 questions. Each carries 10 marks.

34. Sketch the curves given by the equation $x^2y = c$.
Find the slope of tangent at any point (x,y) on these curves.
Also find the equation of the orthogonal trajectories of the family $x^2y = c$.

35. a) Find the length of the curve $\mathbf{r}(t) = \cos 2t\hat{\imath} + \sin 2t\hat{\jmath} + 8t\hat{k}$, $\quad 0 \leq t \leq 2\pi$.
b) Find the maximum directional derivative at (1,1,1) on the surface $x^3(y+z) + y^2z^2 = 3$.

36. Verify the Stoke's theorem for $\mathbf{F} = [y^3, -x^3, 0]$ and S is $x^2 + y^2 \leq 1$, $z = 0$.

(2 × 10 = 20 **Marks.**)