D3BMT2101   (PAGES 2)   Reg.No............................

Name: ....

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2022
(Regular/Improvement/Supplementary)
MATHEMATICS
GMAT3B03T: GEOMETRY AND VECTOR CALCULUS

Time: 2 ½ Hours   Maximum Marks: 80

SECTION A: Answer the following questions. Each carries *two* marks.
(Ceiling 25 Marks)

1. Find the foci and vertices of the ellipse $\frac{x^2}{4} + \frac{y^2}{25} = 1$.

2. Find $\frac{d^2y}{dx^2}$ if $x = \sqrt{t}; y = t$.

3. Find an equation of the tangent line to the curve $x = e^{2t}; y = \log t$ at t=1.

4. Plot the point $(2, \frac{\pi}{2})$ in the $r\theta$ −plane.

5. Find the area of the region enclosed by the lemniscate $r^2 = 4\cos 2\theta$.

6. Find the polar coordinate of the point (-1, 1) in Cartesian coordinate.

7. Find the angle between the 2 planes defined by
$$3x - y + 2z = 1 \text{ and } 2x + 3y - z = 4$$

8. Find the distance between the point (3,1,2) and the plane $2x - 3y + 4z = 7$.

9. Sketch the graph of the cylinder $y^2 + z^2 = 1$.

10. Find an equation in spherical coordinates for the paraboloid with rectangular equation $4z = x^2 + y^2$.

11. Find the domain of the vector function $r(t) = ti + \frac{1}{t}j$.

12. Find the speed of the particle with position vector $r(t) = e^t i + e^{-t}j$ at t=0.

13. Find T(t), where $r(t) = ti + 2t^2 j$ at t=1.

14. Find the domain of the function $f(x, y) = x + 3y − 1$.

15. Determine whether the function $f(x, y) = \frac{2xy}{2x+3y-1}$ is continuous.

SECTION B: Answer the following questions. Each carries *five* marks.
(Ceiling 35 Marks)

16. Find the points on the cardiod $r = 1 + cos\theta$ where the tangent lines are horizontal and where the tangent lines are parallel.

17. Find the area of the region that lies outside the curve $r = 1 − sin\theta$ and inside the curve r=1.

18. Find the equation of the plane containing the points $(3, −1,1); (1,4,2)$ *and* $(0,1,4)$

19. Find the parametric equation and symmetric equation for the line passing through the points (-3,3,2) and (2,-1,4).

20. Find the parametric equation for the tangent line to the helix with parametric equation x = 3 cost; y=2 sint; z=t at $t = \frac{\pi}{2}$.

21. Find the curvature of a circle of radius a.

22. Find the centre, vertices, foci and asymptotes of the hyperbola $3x^2 − 4y^2 − 8y − 16 = 0$.

23. Describe the level curves of the function f(x,y,z) = 2x+4y-3z+1.

(PTO)

SECTION C: Answer any *two* questions. Each carries *ten* marks.

24. (a) Let P be a fixed point on the rim of a wheel. If the wheel has radius a and is allowed to roll along the X- axis without slipping, find the parametric equation of the cycloid.

    (b) Find the length of the curve defined by the parametric equation $x = 2t^2; y = 3t^3; 0 \leq t \leq 1$

25. (a) Identify and sketch the surface $12x^2 - 3y^2 + 4z^2 + 12 = 0$.

    (b) Find the parametric equation for the line of intersection of the planes defined by $3x - y + 2z = 1 \; and \; 2x + 3y - z = 4$.

26. (a) Find a vector function that describes the curve of intersection of the cylinder

    $4z = x^2 + y^2 = 4$ and the plane $x + y + 2z = 4$

    (b) Find the arc length function s(t) for the curve r(t)= 4sin**i**+ 4cost **j** + 3t **k**; t≥0

27. (a) Show that the function $u(x, y) = e^x cosy$ is harmonic in the xy - plane.

    (b) Find the differential of the function $z = \frac{x+y}{x-y}$.

    (2 × 10 = 20 Marks)