

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2022

(Regular/Improvement/Supplementary)

MATHEMATICS: COMPLEMENTARY COURSE FOR PHYSICS, CHEMISTRY & C S

GMAT3C03T: MATHEMATICS - 3

Time: 2 Hours

Maximum Marks: 60

SECTION A: Answer the following questions. Each carries *two* marks.

(Ceiling 20 Marks)

1. Define order of a differential equation. Solve $y' = \cos \pi x$.
2. Solve $y' \sin \pi x = y \cos \pi x$.
3. Write the standard form of a second order linear differential equation. Also state the Fundamental Theorem for the Homogeneous Linear ODE.
4. Solve $y'' - 6y' + 9y = 0$.
5. Solve $y'' + 0.4y' + 9.04y = 0$.
6. Let $a = [1, 1, 1]$, $b = [2, 3, 1]$, $c = [-1, 1, 0]$. Find the angle between: $a - c$ and $b - c$.
7. Find the length of the curve $r(t) = [2 \cos t, 2 \sin t, 6t]$ from $(2, 0, 0)$ to $(2, 0, 24\pi)$.
8. Find $\text{div} \vec{F}$ and $\text{curl} \vec{F}$ where $\vec{F} = \text{grad}(x^2 + y^2 - 3xy)$.
9. Define gradient of a scalar field and also find $\text{grad}(\sqrt{x^2 + y^2 + z^2})$.
10. Explain the physical interpretation of curl.
11. Write a parametric representation of a sphere.
12. Define the terms smooth surface, surface normal.

SECTION B: Answer the following questions. Each carries *five* marks.

(Ceiling 30 Marks)

13. Solve $\frac{dy}{dx} + \frac{x}{1-x^2}y = x\sqrt{y}$.
14. Solve $(\cos x \tan y + \cos(x+y))dx + (\sin x \sec^2 y + \cos(x+y))dy = 0$.
15. Solve $y'' + 4y' + 5y = 25x^2 + 13 \sin 2x$.
16. Solve $y'' + 4y = 4 \sec^2 2x$.
17. Prove that $\text{curl}(\text{grad} f) = \vec{0}$ and $\text{div}(\text{curl} \vec{v}) = 0$
18. Verify Green's theorem for $F_1 = y^2 - 7y$, $F_2 = 2xy + 2x$ where C is the circle $x^2 + y^2 = 1$.
19. Compute the flux of water through the parabolic cylinder S: $x + y + z = 4$, $x \geq 0$, $y \geq 0$, $z \geq 0$, if velocity is $\vec{F} = [x^2, y^2, z^2]$.

(PTO)

SECTION C: Answer any one question. Each carries ten marks.

20. a. If $\vec{v} = 3x^2y^2z^4\mathbf{i} + 2x^3yz^4\mathbf{j} + 4x^3y^2z^3$, find its scalar potential.

b. Prove that $\text{curl}(f\vec{v}) = \nabla f \times \vec{v} + f\text{curl}\vec{v}$

21. a. State Gauss divergence theorem, and use it to evaluate $\iint_S \vec{F} \cdot \vec{n} dA$, where $\vec{F} = [x, y, z]$ and S the sphere $x^2 + y^2 + z^2 = 9$.

b. State Stokes theorem and Calculate this line integral by Stokes's theorem, clockwise as seen by a person standing at the origin, for the following F and C. Assume the Cartesian coordinates to be right handed where $\vec{F} = [4z, -2x, 2x]$, C is the intersection of $x^2 + y^2 = 1$ and $z = y + 1$

(1 × 10 = 10 Marks)