D3BEM2105

Reg.No............................

Name: ....

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2022

(Regular/Improvement/Supplementary)

ECONOMICS & MATHEMATICS (DOUBLE MAIN)

GDMT3B03T: MULTIVARIABLE CALCULUS

Time: 2 ½ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries *two* marks. (Ceiling 25 Marks)

1. Determine where the function is continuous $f(x,y) = \frac{1}{y-x^2}$.
2. Find f_x if $f(x,y,z) = x^2y + y^2z + xz$.
3. Find $\frac{dy}{dx}$ if $x^3 + xy + y^2 = 4$.
4. Define Directional Derivative of a function of two variables $f(x,y)$.
5. What are the steps for finding absolute extremum values of f on a closed bounded set D.
6. State Lagrange's theorem.
7. Define gradient of a function of three variables.
8. Evaluate the iterated integral $\int_0^1 \int_0^{\sqrt{1-y^2}} x dx\, dy$.
9. Sketch the region of integration associated with the interval $\int_0^{\pi} \int_1^4 f(r\cos\theta, r\sin\theta)\, r dr d\theta$.
10. Evaluate the iterated integral $\int_0^1 \int_0^x \int_0^{x+y} x\, dz\, dy\, dx$.
11. Find the Jacobian of the transformation T defined by the equations
    $x = 2u + v$, $y = u^2 - v$.
12. Find the gradient vector field of $f(x,y,z) = x^2 + xy + y^2z^3$.
13. A vector field **F** in R^2 is defined by $\mathbf{F}(x,y) = -x\mathbf{i} - y\mathbf{j}$. Describe **F** and sketch a few vectors representing the vector field.
14. Find the curl of $\mathbf{F}(x,y,z) = x\mathbf{i} + y\mathbf{j}$.
15. Determine whether the vector field $\mathbf{F}(x,y) = (y^2 \cos x)\mathbf{i} + (2y \sin x + 3)\mathbf{j}$ is conservative.

SECTION B: Answer the following questions. Each carries *five* marks. (Ceiling 35 Marks)

16. Let $w = 2x^2y$, where $x = u^2 + y^2$ and $y = u^2 - v^2$. Find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$.
17. Find the domain and range of the function $f(x,y) = \sqrt{4 - x^2 - y^2}$.

18. Let $f(x, y) = x^2 - y^2$. Find the level curve of f passing through the point (5,3). Also find the gradient of f at that point, and make a sketch of both the level curve and the gradient vector.

19. Use a double integral to find the area enclosed by one loop of the three leaved rose $r = \sin 3\theta$.

20. Evaluate $\iiint_B (x^2y + yz^2)dV$ where $B=\{(x, y, z)|-1 \leq x \leq 1, 0 \leq y \leq 3, 1 \leq z \leq 2\}$

21. Let T be the transformation defined by the equations $x = u + v \quad y = u$. Find the image of the rectangular region $S = \{(u, v)|0 \leq u \leq 2, 0 \leq v \leq 1)$ under the transformation T.

22. Find the centroid of a homogeneous solid hemisphere of radius a.

23. Let $\mathbf{F}(x, y) = (2xy^2 + 2y)\,\mathbf{i} + (2x^2y + 2x)\mathbf{j}$

a) Show that F is conservative and find a potential function f such that $\mathbf{F} = \nabla \mathbf{f}$.

b) Use the above result to evaluate $\int_C \mathbf{F}.d\mathbf{r}$ where C is any path from A(−1,1) to B(1,2).

SECTION C: Answer any *two* questions. Each carries *ten* marks.

24. For the following functions, compute $\Delta z, dz$ and compare the values of Δz and dz

(i)$z = 2x^2 + 3y^2$, and suppose that (x, y) changes from (2, -1) to (2.01, -0.98)

(ii)$z = x^2 - 2xy + 3y^2$, and suppose that (x,y) from (2,1) to (1.97,1.02)

25. Find the maximum and minimum values of the function $f(x, y, z) = 3x + 2y + 4z$ subject to the constraints $x - y + 2z = 1$ and $x^2 + y^2 = 4$.

26. Evaluate $\iint_R (2x - y)dA$, where R is the region bounded by the parabola $x = y^2$ and the straight line $x - y = 2$.

27. Let $\mathbf{F}(x, y) = -\frac{1}{8}(x - y)\mathbf{i} - \frac{1}{8}(x + y)\mathbf{j}$ be a force field. Find the work done on a particle that moves along the quarter-circle of radius 1 centred at the origin:

a) in a counter clockwise direction from(1,0)to (0,1).

b) in a clockwise direction from(0,1)to (1,0).

(2 x 10 = 20 Marks)