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**D3BEM2102** (PAGES 2) **Reg.No...........................**

**Name: ..........................**

# THIRD SEMESTER UG DEGREE EXAMINATION, NOVEMBER 2022

**(Regular/Improvement/Supplementary)**

**ECONOMICS & MATHEMATICS (DOUBLE MAIN)**

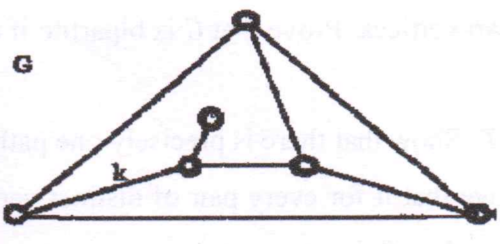
**GDMT3A01T: BASIC LOGIC, BOOLEAN ALGEBRA AND GRAPH THEORY**

**Time: 2 ½ Hours** **Maximum Marks: 80**

## SECTION A: Answer the following questions. Each carries *two* marks.

**(Ceiling 25 Marks)**

1. What do you mean by a proposition? Give examples and non- examples.
2. Verify $p \wedge p \equiv p$.
3. Negate the proposition, where x is an arbitrary integer $(\forall x)(x^2 > 0)$.
4. Prove that there is a positive integer that can be expressed in two different ways as the sum of two cubes.
5. What are the atoms of the lattice N of natural numbers ordered by divisibility?
6. Let A be any nonempty set and let P(A) be the power set of A ordered by set inclusion. What are the first and last elements of P(A)?
7. Draw the Hasse diagram of A = {1, 2, 3, 4, 6, 8, 9, 12, 18, 24} ordered by the relation "x divides y."
8. What is meant by a simple graph? Give an example.
9. Draw one 4-regular graph.
10. State true or false: A complete graph is a regular graph. Justify your claim.
11. Define the intersection of two subgraphs of a graph and give one example.
12. Define a cut vertex. Give one example.
13. Define Euler tour. Give one example
14. Find the number of faces of the given graph



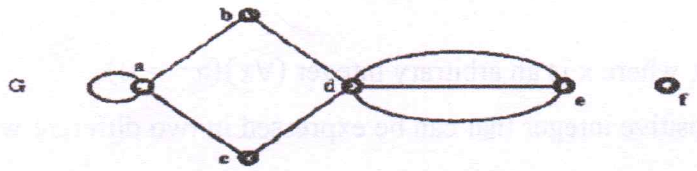

15. Define a planar graph. Give one example.

**(PTO)**

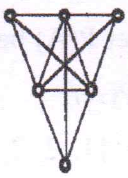
**SECTION B: Answer the following questions. Each carries *five* marks.**

**(Ceiling 35 Marks)**

16. Rewrite each proposition symbolically, where UD= set of all real numbers.
    (i) For all integers x and y, xy=yx
    (ii) There are integers x and y such that x+y=5
17. Prove indirectly: If the square of an integer is odd, then the integer is odd.
18. Let L be a complemented lattice with unique complements. Then show that the join irreducible elements of L, other than 0, are its atoms.
19. State and prove the idempotent law for lattices.
20. Prove that it is impossible to have a group of nine people at a party such that each one knows exactly five of the others in the group.
21. List the degrees of each of the vertices in the following graph $G$. Also specify the number of odd and even vertices of $G$




22. Find $k(G)$for the following graph



23. Let $G$ be a simple graph with $n$ vertices and let $u$ and $v$ be non-adjacent vertices in $G$ such that $d(u) + d(v) \geq n$. Let $G + uv$ denote the supergraph of $G$obtained by joining $u$ and $v$ by an edge. Prove that G is Hamiltonian iff $G + uv$ is Hamiltonian.

**SECTION C: Answer any *two* questions. Each carries *ten* marks.**

24. Construct truth table for each proposition $(i)(p \wedge q) \rightarrow \sim p$ $(ii)(p \vee q) \rightarrow (p \wedge q)$
25. Define consistent enumeration of a finite poset and prove that there exists a consistent enumeration for any finite posetA.
26. Let $G$ be a nonempty graph with at least two vertices. Prove that $G$ is bipartite if and only if it has no odd cycles.
27. a) Let u and v be distinct vertices of a tree $T$. Show that there is precisely one path from u to v
    b) Let $G$ be a graph without any loops. Prove that if for every pair of distinct vertices u and v of $G$ if there is precisely one path from u to v then $G$ is a tree.

**(2 x 10 = 20 Marks)**