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D2BMT2302 (Pages: 2) Name:.....

Reg. No:.....

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2024

(Regular/Improvement/Supplementary)

MATHEMATICS: Complementary Course for Physics, Chemistry and CS

GMAT2C02T: MATHEMATICS - 2

Time: 2 Hours **Maximum Marks: 60**

SECTION A: Answer the following questions. Each carries *Two* marks.

(Ceiling 20 Marks)

1. Find $\frac{dy}{dx}$ where $y = 6\sinh\frac{x}{3}$.
2. Show that $\cosh^2 x - \sinh^2 x = 1$.
3. Evaluate $\int \sinh\frac{x}{5}dx$.
4. Plot the point $(-2, \frac{\pi}{2})$ given in polar coordinate.
5. Find an equation for the hyperbola with eccentricity $\frac{3}{2}$ and directrix $x = 2$.
6. Replace the cartesian equation $x^2 + (y-2)^2 = 4$ by equivalent polar equation.
7. Describe the level surfaces of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$.
8. Calculate the value of f at $z = 1 + 3i$ where $f(z) = z^2 + 3z$.
9. State Liouville's theorem.
10. Find the real and imaginary part of $\frac{z_1}{z_2}$ where $z_1 = 4 + 3i$ and $z_2 = 2 - 5i$.
11. Evaluate $\int_{-\pi i}^{\pi i} \cos z dz$.
12. State Cauchy's integral formula.

SECTION B: Answer the following questions. Each carries *five* marks.

(Ceiling 30 Marks)

13. (a) Show that the function $f(x, y) = \frac{2x^2y}{x^4+y^2}$ has no limit as (x,y)approaches (0,0).

    (b) Find $\lim_{(x,y)\to(0,0)} \dfrac{e^y \sin x}{x}$.

(PTO)

14. Find all second order partial derivatives of the function $f(x,y) = xe^y + y + 1$.

15. Determine the linearization of $f(x,y) = x^2 - xy + \frac{1}{2}y^2 + 3$ at the point (3,2).

16. Find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ where $w = xy + yz + xz$, $x = u + v$, $y = u - v$ and $z = uv$.

17. Evaluate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at (1,1,1) if $z^2 - xy + y^2z + y^4 = 0$.

18. Represent $\frac{1-i}{1+i}$ in polar form.

19. Verify that $u = x^2 - y^2 - y$ is harmonic in the whole complex plane and find a conjugate harmonic function v of u.

SECTION C. Answer any *One* question. Each carries *Ten* marks.

20. (a) Find the area of the region that lies inside the circle $r = 1$ and outside the cardioid $r = 1 - \cos\theta$.

    (b) Determine the slope of the curve $r = 1 - \cos\theta$ at $\theta = \frac{\pi}{2}$.

21. Evaluate

    (a) $\oint_C \frac{1}{z^2+1}\,dz$ where $C: |z+i| = 1$, counterclockwise.

    (b) Integrate $f(z) = z^{-2}\tan\pi z$ counter clockwise where C is any contour enclosing 0.

(1 x 10 = 10 Marks)