D2BMT2301 (PAGES 2) **Reg.No.....**

Name: .....

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2024

(Regular/Improvement/Supplementary)

MATHEMATICS

GMAT2B02T: CALCULUS AND INFINITE SERIES

Time: 2 ½ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries *two* marks.

(Ceiling 25 Marks)

1. Find the inverse of the function $f(x) = \frac{2x-3}{5x-7}$ where the domain of f excludes $x = \frac{7}{8}$.
2. Solve $e^{2x+3} = 10$
3. Evaluate $\int e^{5x}\, dx$
4. Differentiate $y = 3^{(x+1)}$
5. Evaluate the expression $\log_3 \frac{1}{81}$.
6. Show that derivative of $\sinh x$ is $\cosh x$.
7. Express the number $coth^{-1}(\frac{5}{4})$ in terms of natural logarithms.
8. Evaluate $\lim_{x\to 0} \frac{e^x-1}{2x}$.
9. Write the first five terms of the sequence $a_n = \frac{n-1}{n^2+1}$.
10. State Squeeze Theorem for Sequences.
11. Discuss the convergence of geometric series.
12. Find the Maclaurin series of $f(x) = \cos x$.
13. Determine the radius of convergence of $\sum_{k=0}^{\infty} \frac{k^5}{(k+1)!}\, x^k$.
14. Show that the series $1 + \frac{1}{2} + \frac{1}{2^2} + \cdots$. converges and also find its sum.
15. What is an alternating series? Give an example.

SECTION B: Answer the following questions. Each carries *five* marks.

(Ceiling 35 Marks)

16. Find the area of the region between the graphs of $y = x^2 + 2$ and $y = x - 1$ and the vertical lines $x = -1$ 1 and $x = 2$.
17. Find the length of the graph of f(x)= $(x-1)^{\frac{3}{2}} + 2$ on [1, 2].
18. State and prove power rule.

(PTO)

19. Show that $\lim_{n\to\infty}(1+h)^{\frac{1}{h}} = e$

20. Evaluate $\int_2^4 \frac{1}{\sqrt{4-x}}\, dx$.

21. Prove that $\lim_{n\to\infty}|a_n| = 0$ then the sequence (a_n) converges to zero.

22. Justify the statement : Every conditionally convergent series need not be absolutely convergent.

23. Find the Taylor series for $f(x) = In\ x$ at 1 and determine its interval of convergence.

SECTION C: Answer any *two* questions. Each carries *ten* marks.

24. A solid has a circular base of radius 2. Parallel cross sections of the solid perpendicular to its base are equilateral triangles. What is the volume of the solid?

25. a) Show that $\frac{d}{dx} sinh^{-1}x = \frac{1}{\sqrt{x^2+1}}$

b) The concentration of a certain drug (in mg/cc) in a patient's bloodstream t hr after injection is $C(t) = \frac{t}{5(t^2+1)}$. Determine the average concentration of the drug in the patient's bloodstream over the first 4 hr after the drug is injected.

26. State and Prove ratio test for convergence of series.

27. a) State limit comparison test and hence show that the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+1}$ divergent.

b) Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}2n}{4n-1}$ converges or not.

(2 x 10 = 20 Marks)