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D2BMT2001

Reg.No.............................

Name: ....

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2021

MATHEMATICS

GMAT2B02T - CALCULUS AND INFINITE SERIES

Time: 2.5 Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries *two* marks
(Ceiling 25)

1. Express the volume of the solid obtained by revolving the region under the graph of $y = \sqrt{x}$ on $[0,2]$ about the x axis as an integral.
2. Write the integral that gives the arc length of (1) a smooth function $y = f(x)$ on the interval $[a, b]$ (2) a smooth function $x = g(y)$ on the interval $[c, d]$.
3. Define a solid of revolution. Write the integral that gives the volume of a solid of revolution using the disk method where the axis of revolution is x axis.
4. Prove that $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$.
5. Find $\int x \sec x^2 \, dx$.
6. Solve the equation $2e^{x+2} = 5$.
7. Evaluate $\int_0^1 3^x \, dx$.
8. Find $\cot\left(\sin^{-1}\frac{1}{3}\right)$.
9. Define a divergent sequence. Give an example.
10. Find $\lim\limits_{n\to\infty} e^{\sin(1/n)}$.
11. Determine whether the series $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ is divergent or not.
12. State the Limit Comparison Test
13. Find the radius of convergence and interval of convergence of $\sum_{n=0}^{\infty} n!\, x^n$.
14. Determine whether the series $\sum_{n=1}^{\infty}(-1)^n \frac{2n}{4n-1}$ converges or diverges.
15. Define a power series in x.

SECTION B: Answer the following questions. Each carries *five* marks
(Ceiling 35)

16. Find the area of the region between the graphs of $y = x^2 + 2$ and $y = x - 1$ and the vertical lines $x = -1$ and $x = 2$.
17. Find the arc length of the graph of the equation $y = 2(x-1)^{3/2}$ from $P(1,0)$ to $Q(5,16)$.
18. Evaluate $\int \frac{1}{x\sqrt{x^4 - 16}} dx$.
19. Find the derivative of $y = \frac{(2x-1)^3}{\sqrt{3x+1}}$.

20. State Squeeze theorem for Sequences. Applying Squeeze theorem, show that

$$\lim_{n\to\infty} \frac{n!}{n^n} = 0.$$

21. Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{3+2^n}$ converges or diverges.

22. Determine whether the series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n^2+1}{2^n}$ is absolutely convergent, conditionally convergent or divergent.

23. Find the Taylor series representation of $f(x) = \frac{1}{1+x}$ at $x = 2$.

SECTION C: Answer any *two* questions. Each carries *ten* marks (2 × 10 = 20 Marks)

24. (a) Find the area of the surface obtained by revolving the graph of $x = y^3$ on the interval $[0,1]$ about the y axis.

(b) A solid has a circular base of radius 2. Parallel cross sections of the solid perpendicular to its base are equilateral triangles. What is the volume of the solid.

25. (a) Evaluate $\lim_{x\to 0^+} \left(\frac{1}{x}\right)^{\sin x}$.

(b) Find the derivative of $f(x) = x^x$.

26. (a) Determine whether the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ is convergent or divergent.

(b) Determine whether the series $\sum_{n=1}^{\infty} \frac{\sqrt{n}+\ln n}{n^2+1}$ converges or diverges.

27. (a) Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} 3.5.7,,,(2n+1)}{1.4.7\ldots(3n-2)}$ is convergent, absolutely convergent, conditionally convergent or divergent.

(b) Find the Maclaurin series of $f(x) = e^x$ and determine its radius of convergence.