

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2024
(Regular/Improvement/Supplementary)
STATISTICS: COMPLEMENTARY COURSE FOR MATHEMATICS & CS
GSTA2C02T: PROBABILITY THEORY

Time: 2 Hours

Maximum Marks: 60

SECTION A: Answer the following questions. Each carries *two* marks.

(Ceiling 20 Marks)

1. What is sample space? A coin is tossed until a head appears, write down the sample space.
2. State empirical definition of probability.
3. A problem is given to two students and their chances of solving it are $1/2$ and $1/3$ respectively. What is the probability that the problem will be solved?
4. A can hit a target four times in 5 shots, B, three times in 4 shots, C two times in 3 shots. Calculate the probability that only one will hit the target.
5. Let X be a random variable with p.d.f $f(x) = kx^2(1-x); 0 < x < 1$. Find the value of k.
6. If X has the pdf $f(x) = \begin{cases} 1, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$
Obtain the distribution of $-2\log X$.
7. If $f(x) = \frac{1}{2^x}, x = 1, 2, 3, \dots$
Show that $E(2^X)$ does not exist.
8. What are the properties of moment generating function?
9. What do you mean by conditional probability function.
10. For any two random variables X and Y, show that $E(E(X|Y)) = E(X)$.
11. If X is a random variable with pdf f(x), Prove that $E(X^2) \geq [E(X)]^2$.
12. What are the properties of distribution function?

SECTION B: Answer the following questions. Each carries *five* marks.

(Ceiling 30 Marks)

13. Twenty-five books are placed at random in a shelf. Find the probability that a particular pair of books shall be
 - (i) Always together
 - (ii) Never together.
14. State and prove addition theorem on probability for two events.

(PTO)

15. Find p.d.f of a random variable with distribution function $F(x) = 1 - e^{-x}$, $x > 0$.
16. Define raw moments and central moments. Obtain the relation between raw moments and central moments.
17. Two dice are thrown. X represents the sum of the two numbers that come up. Determine $E(X)$ and $V(X)$.
18. Find the mean of $Y = X^2 + 1$ if X has probability function.

x :	0	1	2	3
P(x) :	0.1	0.2	0.3	0.4

19. Find the m.g.f for $f(x) = \frac{1}{8}(1+x)$, $2 < x < 4$.

SECTION C: Answer any one question. Each carries ten marks.

20. (a) State and prove Bayes' theorem.
- (b) The probabilities of X, Y and Z becoming managers are $\frac{4}{9}$, $\frac{2}{9}$ and $\frac{1}{3}$ respectively. The probabilities that the Bonus Scheme will be introduced if X, Y and Z becomes managers are $\frac{3}{10}$, $\frac{1}{2}$ and $\frac{4}{5}$ respectively.
- (i) What is the probability that Bonus Scheme will be introduced?
- (ii) If the Bonus Scheme has been introduced, what is the probability that the manager appointed was X?
21. The p.d.f of two random variables (X, Y) is given by $f(x, y) = \begin{cases} 2, & 0 < x < y < 1 \\ 0, & \text{elsewhere} \end{cases}$

Find the marginal distributions. Also find the conditional mean and variance of X given $Y = y$.

(1 x 10 = 10 Marks)