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D2BHM2304 (PAGES 3) Reg.No..............................

Name: ....

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2024

(Regular/Improvement/Supplementary)

HONOURS IN MATHEMATICS

GMAH2B08T: DISTRIBUTION THEORY

Time: 3 Hours **Maximum Marks: 80**

PART A: Answer *all* the questions. Each carries *one* mark.

Choose the correct answer.

1. If X is a random variable, then V(3X +1) = ...

   a) 9V(X) b) 9V(X) + 1 c) 3V(X) + 1 d) 3V(X)

2. X and Y are two independent random variables, V(3X + 4Y) = ...

   a) 3V(X) + 4V(Y) b) 9V(X) + 16 V(Y) c) V(X) + V(Y) d) zero

3. If X follows Binomial distribution B(8, 0.4), then the distribution of Y = 8 –X:

   a) B(8, 0.4) b) B(8, 0.6) c) B(4, 0.4) d) B(4, 0.6)

4. If X follows Uniform over [-1,1], then its mean =........

   a) 0 b) 1 c) 1/3 d) 2

5. Seventh central moment of N(μ, σ) is:

   a) Zero b) One c) σ^7 d) μ^7

Fill in the Blanks.

6. If X is a random variable, then V(6X) = ...
7. If E(X) = 3 and E(X^2) = 19, then V(X)=.........
8. In a Beta distribution of first kind, m=n =1, the distribution reduces to........
9. Name of the continuous distribution with lack of memory property is.........
10. If X is a standard normal variate, then P(X > 3) =......

(10 x 1 = 10 Marks)

PART B: Answer any *eight* questions. Each carries *two* marks.

11.

X :	-3	-2	-1	1	2
f(x):	0.1	0.2	0.4	0.2	0.1

Determine E(X).

12. For a certain distribution $\mu_r' = r!$. Find an expression for the corresponding m.g.f and identify the distribution.

(PTO)

13. Define joint probability mass function and joint probability density function of a bivariate random variable .

14. If X and Y are two independent random variables.

show that $M_{X+Y}(t) = M_X(t).M_Y(t)$.

15. For any two random variables X and Y, show that $E(E(X|Y)) = E(X)$.

16. The mean and variance of Binomial variate X with parameters n and p are 16 and 8 respectively. Find $P(X = 1)$ and $P(X > 2)$.

17. If $X \to N(\mu, \sigma)$, Show that $\mu_{2r+2} = (2r+1)\sigma^2 \mu_{2r}$.

18. If X follows Normal with mean 35 and variance 4. Find.

(i) $P(X < 40)$. (ii) $P(45 < X < 60)$.

19. State Chebyshev's inequality.

20. State central limit theorem.

(8 x 2 = 16 Marks)

PART C: Answer any *six* questions. Each carries *four* marks.

21. Two dice are thrown. X represents the sum of the two numbers that come up. Determine E(X) and V(X).

22. Let X be a r.v with the following distribution:

X :	0	1	2	3	4	5	6
P(x) :	1/20	P_1	1/5	P_2	P_3	1/10	1/10

If $E(X) = 3.1$, $E(X^2) = 12.1$. Find P_1, P_2 and P_3.

23. If $f(x,y) = \frac{1}{252} x^2(y+2); x = 1,2,3$ *and* $y = 1,2,3,4$ is the joint p.m.f of (X,Y).

Find the marginal p.d.f's.

24. Derive Moment generating function of Binomial distribution. Hence show that mean > variance.

25. If $X \to P(\lambda)$. Show that $\mu_{r+1} = \lambda\left[r\,\mu_{r-1} + \frac{d\mu_r}{d\lambda}\right]$

26. Show that exponential distribution does not possess additive property.

27. Find the least value of probability $P[1 \le X \le 7]$, where X is a r. v. with E(X) = 4 and V (X) = 4.

28. (X_k), k = 1, 2, 3, … is a sequence of independent random variables each taking the value -1, 0, 1. Given that $P(X_k = 1) = P(X_k = -1) = \frac{1}{k}$ and $P(X_k = 0) = 1 - \frac{2}{k}$. Examine whether the law of large numbers holds for this sequence.

(6 x 4 = 24 Marks)

PART D: Answer any *two* questions. Each carries *fifteen* marks.

29. The joint p.d.f of X and Y is given by the following table.

Y \ X	1	3	9
2	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$
4	$\frac{1}{4}$	$\frac{1}{4}$	0
6	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$

(i) Find Cov (X, Y). (ii) Are X and Y independent?

30. (i) Prove that $\mu_{r+1} = pq\left[nr\,\mu_{r-1} + \frac{d\mu_r}{dp}\right]$, where μ_{r+1}, μ_{r-1}, μ_r are the central moments of Binomial distribution with parameters n and p. Hence obtain the first four central moments.

(ii) Derive MGF of Binomial distribution.

31. (i) If $X \to N(\mu, \sigma)$, Prove that $\mu_{2r} = 1.3.5\ldots(2r-1)\sigma^{2r}$.

(ii) Using the above result obtain 2nd and 4th central moments. Hence find kurtosis.

(2 x 15 = 30 Marks)